PART 2.

Oscillator of Quasi-periodic Oscillations.

Two-dimensional Torus Doubling Bifurcation

We propose a new autonomous differential dynamical system with dimension $N = 4$, whose solution represents stable two-frequency oscillations. It is shown that the system realizes a sequence of period doubling bifurcations of two-dimensional ergodic tori. It is established that when the doubling bifurcation takes place, no resonances on a torus are observed, the ergodic torus is doubled.

Model of the Oscillator

First consider Anishchenko-Astakhov's oscillator:

$$\begin{aligned} \dot{x} &= mx + y - xz - dx^3, \\ \dot{y} &= -x, \\ \dot{z} &= -gz + g\Phi(x). \end{aligned} \qquad (1)$$

The first two equations describe the Van der Pol oscillator.

The third equation represents an inertial cascade of additional feedback.

$\Phi(x)$ describes a nonlinear convertor and can be defined in the form

(exp(x) – 1) or $I(x)x^2$, where $I(x) = 1$ for $x > 0$ and $I(x) = 0$ for $x \le 0$.

Now we change the inertial cascade of additional feedback that causes the dimension of the equations to increase:

$$\dot{z} = \varphi,$$
$$\dot{\varphi} = -\gamma\varphi + \gamma\Phi(x) - gz. \qquad (2)$$

γ is the parameter of damping of the new filter,

g is the parameter characterizing its inertia.

Equations (2) represent the equation of a dissipative circuit in the regime of forced oscillations:

$$\ddot{z} + \gamma\dot{z} + gz = \gamma\Phi(x). \qquad (3)$$

Our studies have shown that the regime of undamped autonomous oscillations can be obtained if the derivative $\dot{z}(t) = \varphi(t)$ is used instead of the controlling signal $z(t)$.

The new oscillator is defined by the following system of equations:

$$\begin{aligned}
\dot{x} &= mx + y - x\varphi - dx^3, \\
\dot{y} &= -x, \\
\dot{z} &= \varphi, \\
\dot{\varphi} &= -\gamma\varphi + \gamma\Phi(x) - gz.
\end{aligned} \qquad (4)$$

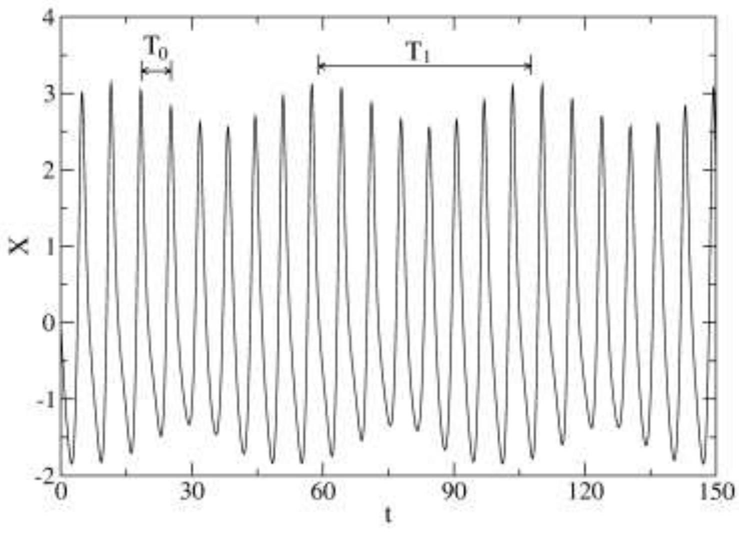
This is a nonlinear dissipative dynamical system of dimension $N = 4$ and is characterized by four controlling parameters.

m is the parameter of excitation,

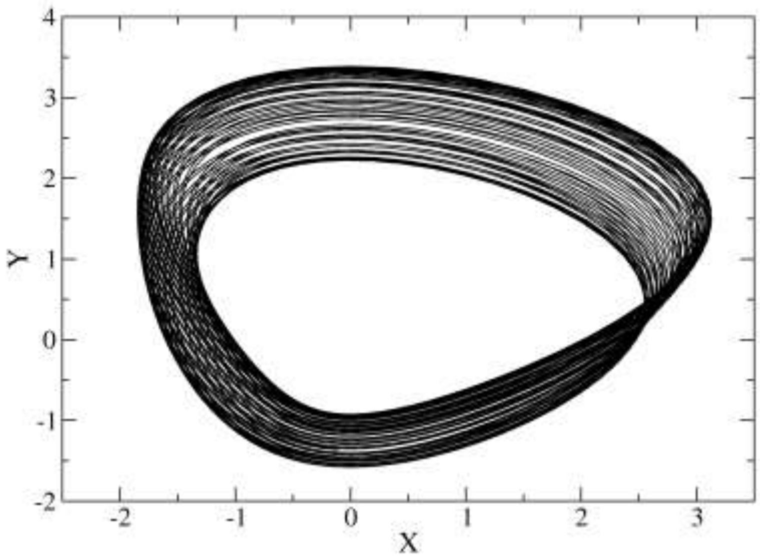
d is the parameter of nonlinear dissipation,

γ is the parameter of damping, and

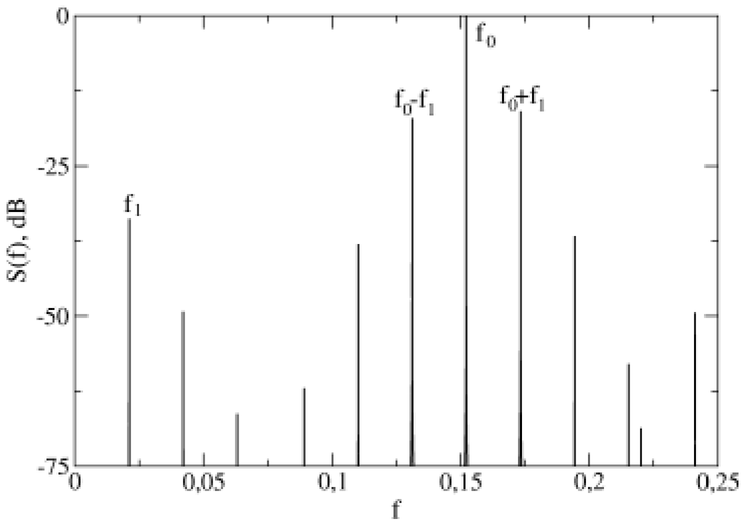
g is the parameter of filter inertia.



(a) - x(t) time realization



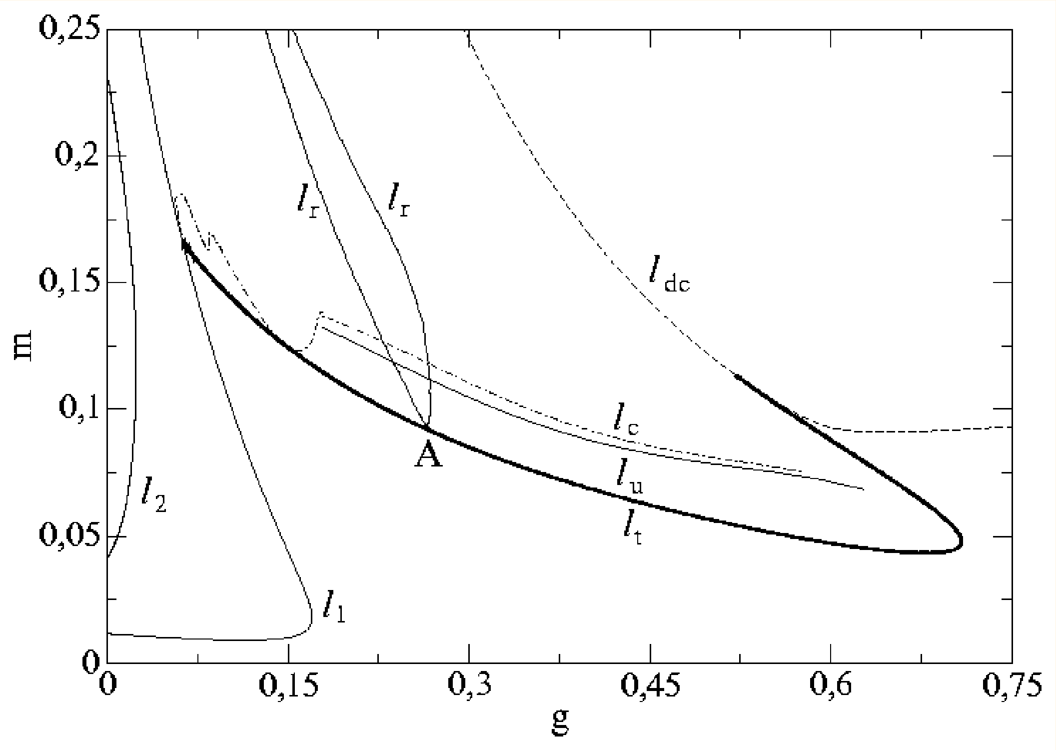
(c) – phase portrait



(b) - power spectrum

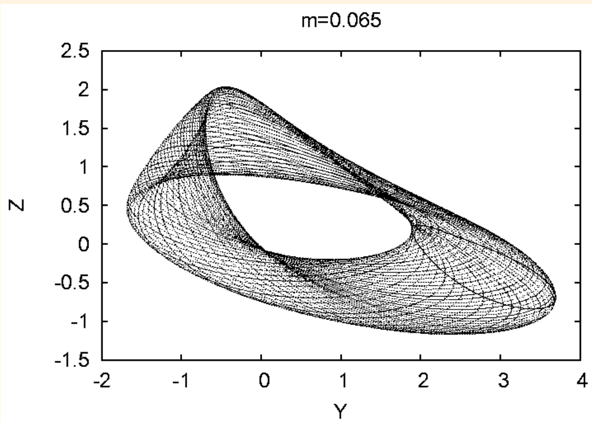
Numeric results

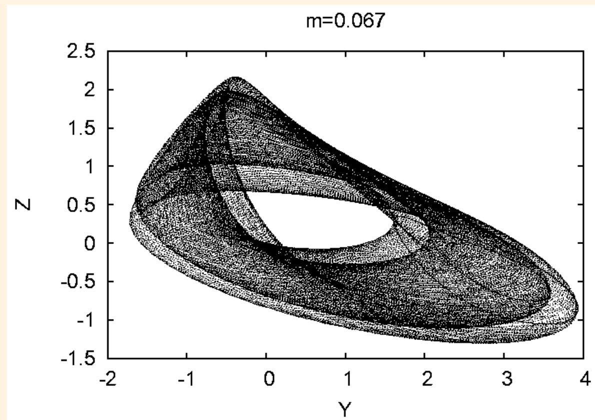
It has been numerically established that system (4) can realize the regime of a stable two-dimensional torus, the torus period doubling bifurcation and the torus breakdown with the transition to chaos.

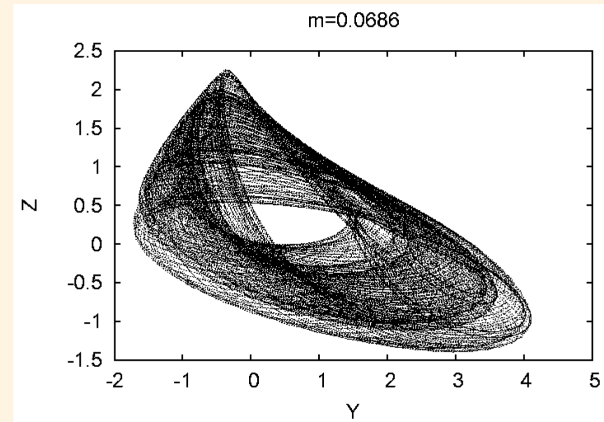


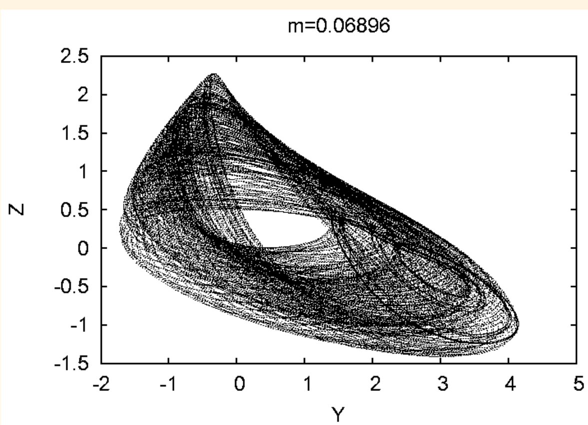
Bifurcation diagram of system modes on the controlling parameter plane for γ = 0.2 and d = 0.001. $\Phi(x)$ is defined in the form $I(x)x^2$.

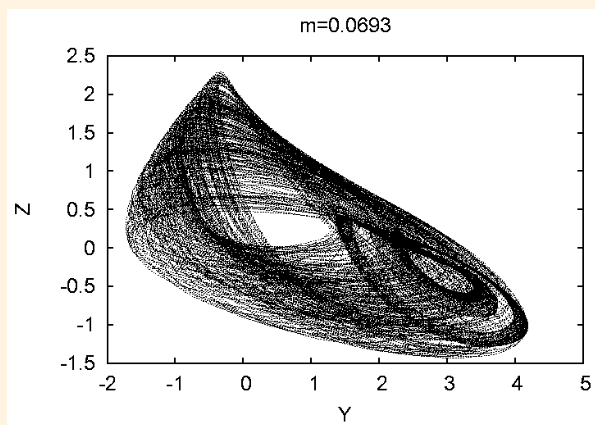
Projections of attractors and the corresponding Poincare sections at the bifurcation points of the 2*D* torus period doubling for g = 0.5, γ = 0.2 and d = 0.001

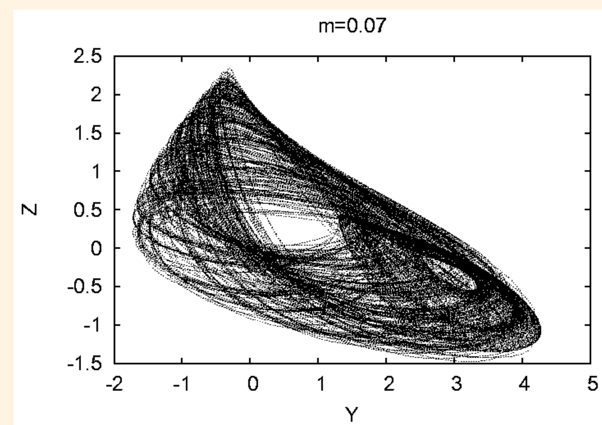




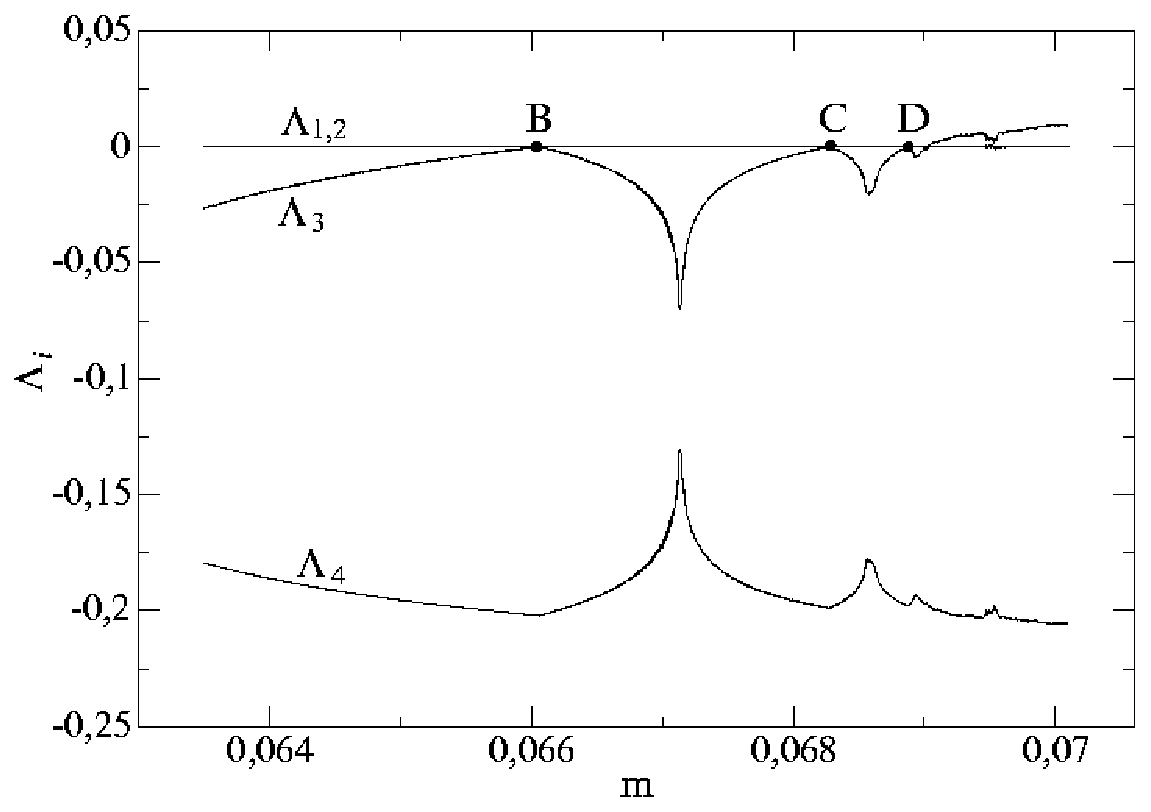








Dependence of the Lyapunov exponent spectrum on the parameter *m* for g = 0.5, γ = 0.2 and d = 0.001. *B, C* and *D* are the bifurcation points of torus period doubling



At the bifurcation transition:

0, 0, --, -- $\Rightarrow$ 0, 0, 0, -- $\Rightarrow$ 0, 0, --, --

torus$_2$ torus$_3$ torus$_2$

External Synchronization of Torus Oscillator

$$\xi(t) = A_0(1 + M\sin(2\pi f_1 t)) \times \cos[2\pi(f_2 + \Delta f)t]$$

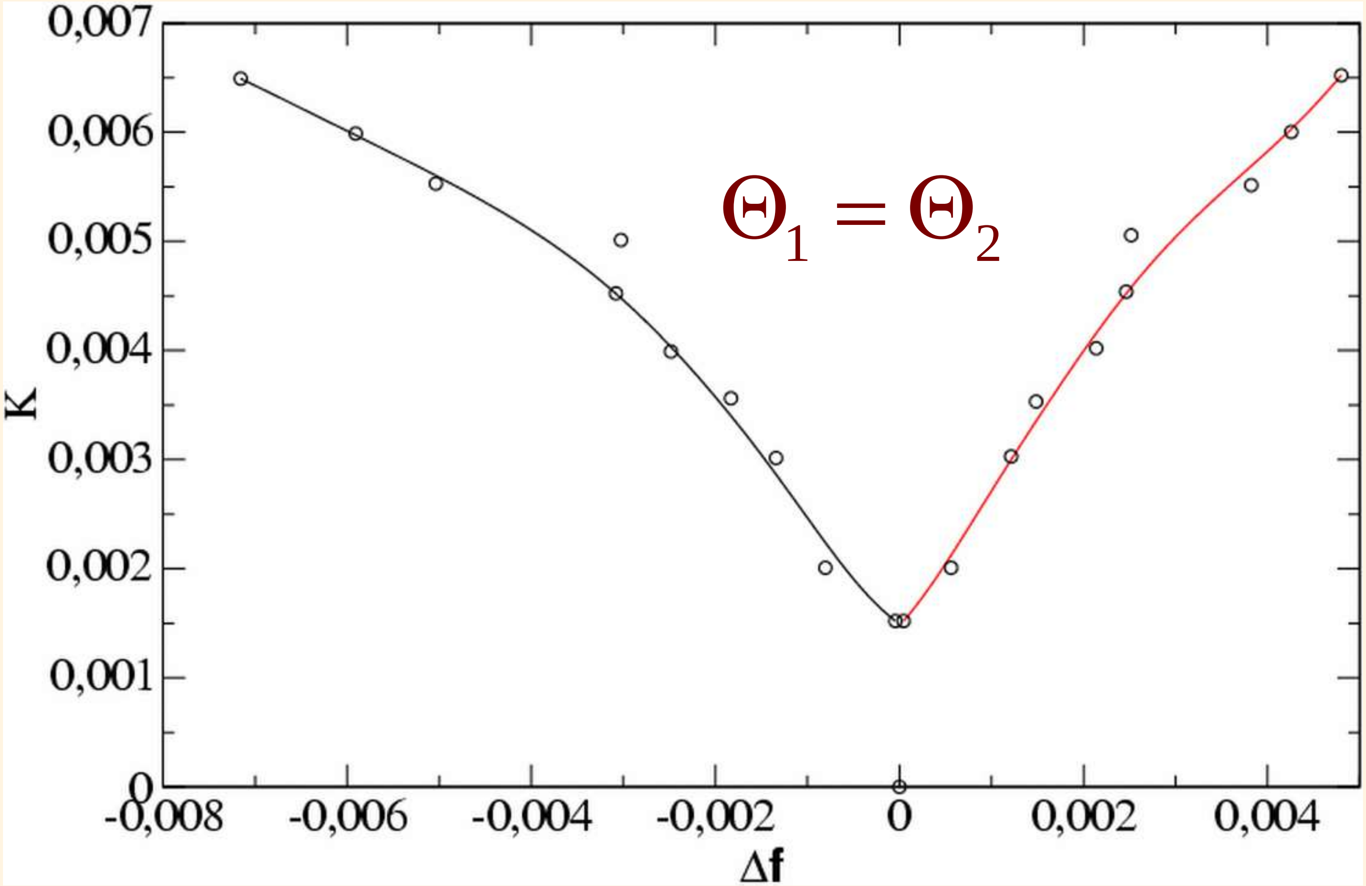
$$\Theta = \frac{f_1}{f_2} = \Theta_1$$

→

Torus Oscillator

$$\Theta = \frac{f_1}{f_0} = \Theta_2$$

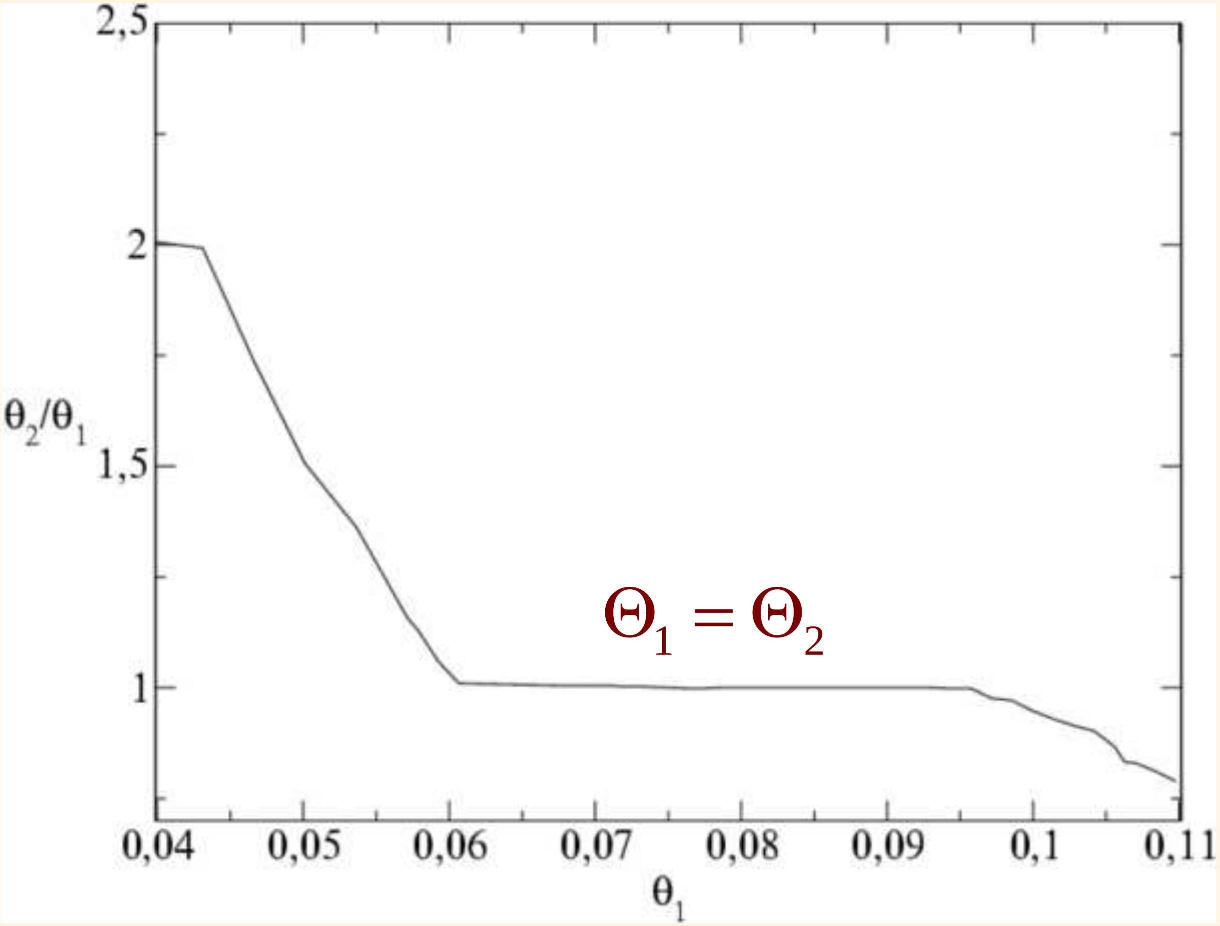
$$\begin{aligned}
\dot{x} &= mx + y - x\varphi - dx^3 + K\xi(t),\\
\dot{y} &= -x,\\
\dot{z} &= \varphi,\\
\dot{\varphi} &= -\gamma\varphi + \gamma F(x) - gz,\\
\xi(t) &= A_0[1 + M\sin(2\pi f_1 t)]\cos[2\pi(f_2 + \Delta f)t].
\end{aligned}$$



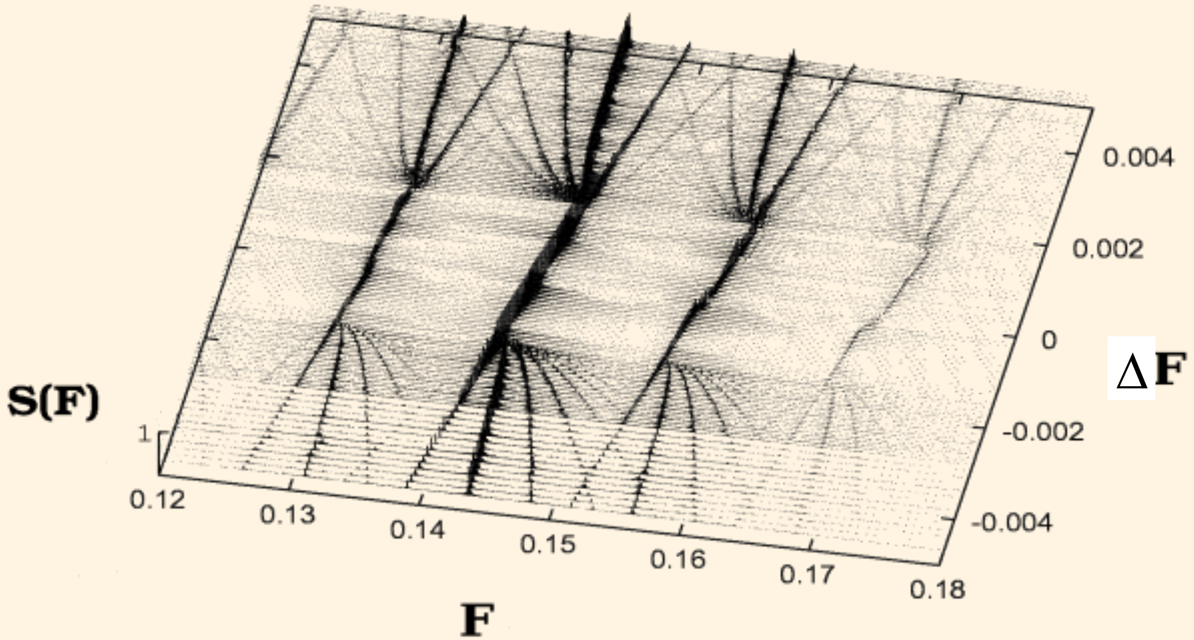
Synchronization region on the plane “frequency mismatch – external force amplitude”.

Effect of winding number locking.

(The winding number is changed by varying the basic frequency)

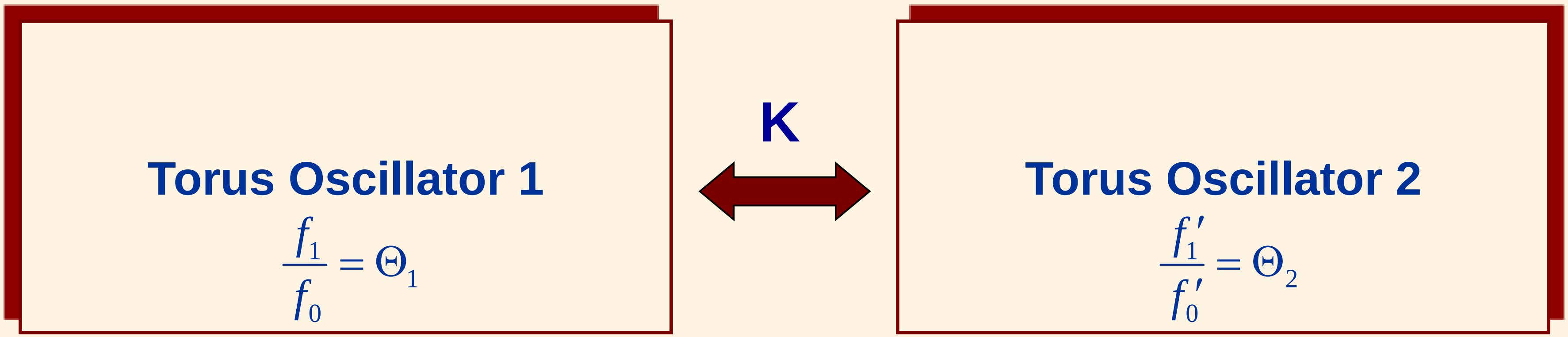


Effect of winding number locking by the external signal as the modulation frequency is varied *(K* = const)*.*



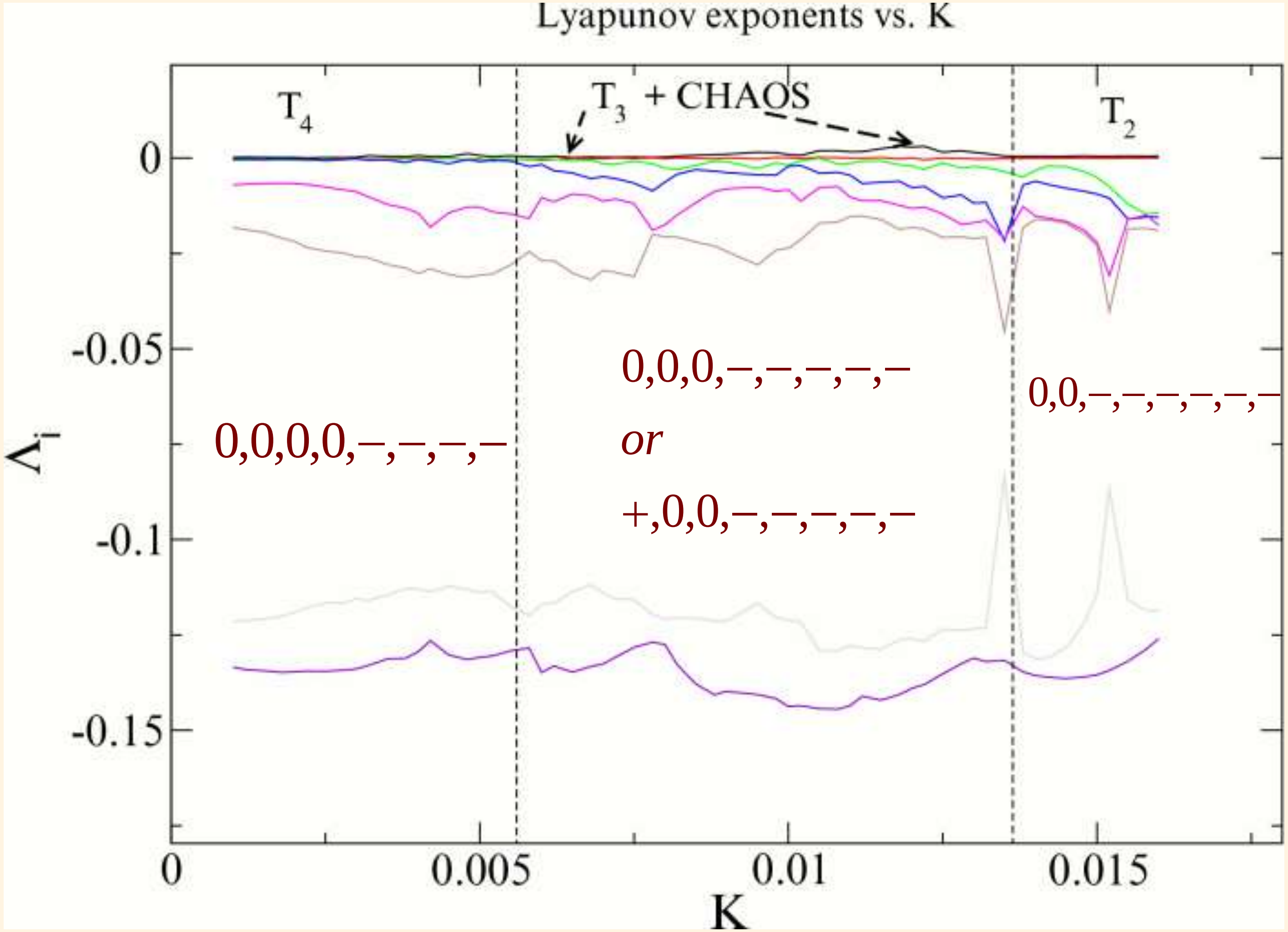
Evolution of the power spectrum for the winding number locking effect as varying basic frequency of the external force.

Mutual Synchronization of Two Coupled Torus Oscillators



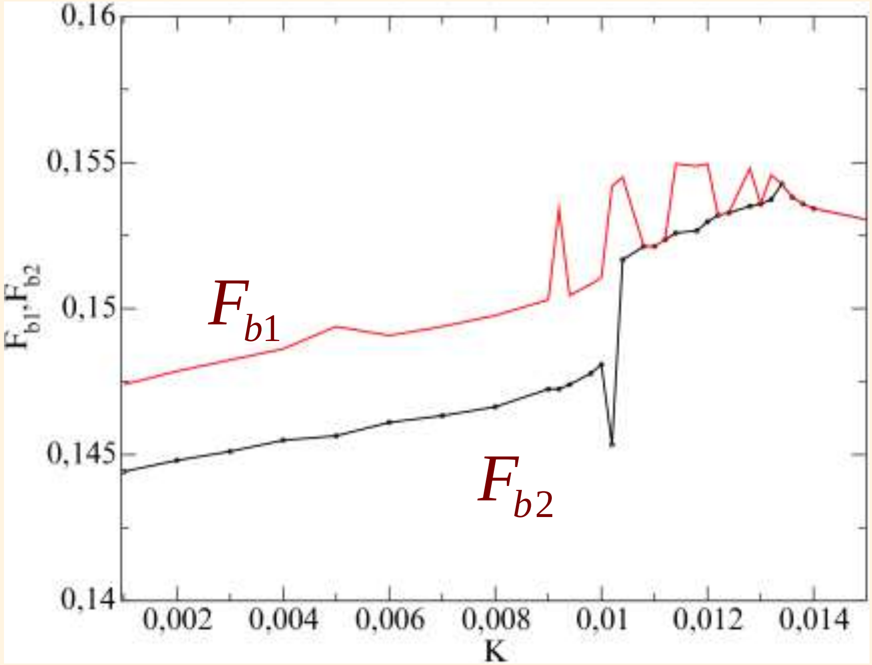
$$
\begin{aligned}
\dot{x}_1 &= mx_1 + y_1 - x_1\varphi_1 - dx_1^3 + K(x_2 - x_1),\\
\dot{y}_1 &= -x_1,\\
\dot{z}_1 &= \varphi_1,\\
\dot{\varphi}_1 &= -\gamma\varphi_1 + \gamma F(x_1) - g_1 z_1,\\
\dot{x}_2 &= mx_2 + y_2 - x_2\varphi_2 - dx_2^3 + K(x_1 - x_2),\\
\dot{y}_2 &= -x_2,\\
\dot{z}_2 &= \varphi_2,\\
\dot{\varphi}_2 &= -\gamma\varphi_2 + \gamma F(x_2) - g_2 z_2.
\end{aligned}
$$

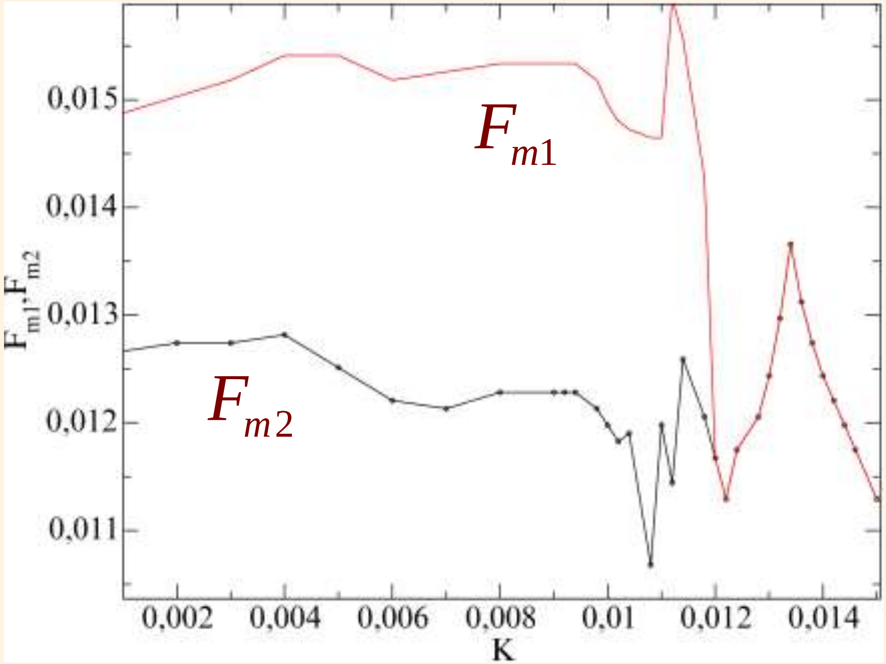
$$m = 0.065, \quad d = 0.001, \quad \gamma = 0.2.$$



Lyapunov exponents spectrum of the two coupled torus oscillators system vs. the coupling strength

In the region T_2 the effect of mutual torus synchronization takes place.





Effect of basic and modulation frequency locking when increasing coupling strength.

For $K \geq 0.14$, the effect of winding number locking is realized.