Reconstruction of dynamic systems as applied to secure communications

V. S. Anishchenko, A. N. Pavlov, and N. B. Yanson

Saratov State University, 410026 Saratov, Russia
(Submitted June 16, 1997)
Zh. Tekh. Fiz. **68**, 1–8 (December 1998)

An analysis is made of a method of reconstructing signals which parametrically modulate a chaotic generator using a one-dimensional realization of its oscillation process. Test examples are used to demonstrate the efficiency of this method for simultaneous independent transmission of several information signals over a single communication channel. © *1998 American Institute of Physics.* [S1063-7842(98)00112-3]

INTRODUCTION

One line of research in modern nonlinear dynamics is the reconstruction of dynamic systems. Interest in this problem over the last seventeen years was stimulated by the appearance of Refs. 1 and 2. Packard *et al.*[1] showed that the phase portrait of the attractor of a dynamic system can be reconstructed from the scalar time series $a_i = a(i\Delta t)$ if data from the same series a_i, taken after some delay, are used as the missing coordinates of the state vector. In Ref. 2 the possibility of reconstructing the phase portrait of an attractor using a one-dimensional time series was given theoretical foundation in the form of the Takens theorem.

The appearance of Ref. 2 was the precursor to using this theorem to predict system behavior,[3,4] and to calculate the metric[5] and dynamic[6] characteristics of an attractor using a time series. In a study published in 1987, Cremers and Hübler[7] proposed a method of reconstructing the equation for a dynamic system using its one-dimensional realization. The idea of this method (global reconstruction method) has been developed in a wide range of studies (for instance, Refs. 8–10).

Despite some differences in the global reconstruction algorithms used by different researchers, they all propose to solve the modeling problem in two stages. The first involves calculation of the embedding space dimension n. After the value of n has been selected, the phase portrait of the dynamic system is reconstructed using the scalar time series. In addition to the delay method,[1,2] any of the methods proposed in Ref. 11 can be used for this purpose. At the second stage of the algorithm, the general form of the mathematical model is defined and the evolution equations specified.

When series differentiation of the initial time series is used to reconstruct the phase portrait,[7–11] the mathematical model has the form

$$\frac{dx_1}{dt} = x_2, \quad \frac{dx_2}{dt} = x_3, \ldots,$$

$$\frac{dx_n}{dt} = f(x_1, x_2, \ldots, x_n, \boldsymbol{\mu}), \tag{1}$$

where f is the nonlinear function which needs to be determined and $\boldsymbol{\mu}$ is the vector of the parameters.

This method of reconstruction envisages that the general form of the function $f(x_1, x_2, \ldots, x_n, \boldsymbol{\mu})$ must be defined *a priori*. In general, even an approximate form of this function is impossible to determine *a priori* and thus, this function is usually represented as an expansion in terms of a certain basis and the expansion coefficients determined numerically. In addition to the standard basis $1, x_1, x_1^2, x_1x_2, \ldots$, the nonlinearity can also be approximated by an expansion in terms of any set of orthogonal basis functions (such as Legendre polynomials). In any case, however, the need to define the form of the function f is a serious disadvantage of this method of reconstructing the dynamic model of a system.

A fundamentally different situation arises when the explicit form of the nonlinear function f is known *a priori* and only the expansion coefficients are unknowns. In this case, the global modeling problem can be solved to a certain accuracy, which is determined by the number of points in the initial time series available for observation, the smallness of the discretization step, and the accuracy of writing the data. In Ref. 12 we showed that a consequence of solving the problem of reconstructing the mathematical model may be interesting applications, one of which is the use of the reconstruction technique to obtain secure communications.

It should be noted that the appearance of wide range of applied problems in modern nonlinear dynamics (including secure communications) in recent years has been stimulated by the development of concepts of dynamic chaos, especially the phenomenon of synchronized chaos. This effect was used by the authors of pioneering work on the confidentiality of transmitted information, using wide-band oscillations of a chaotic generator as a masking[13–15] or carrier signal.[16–18] The first case was based on the principle of the synchronization of chaotic systems proposed in Ref. 19. An alternative method of securing transmitted information involved the experimental control of chaos.[20,21]

Here we propose a new method of solving the problem of secure communications based on the global reconstruction of a dynamic system. Whereas in Ref. 12 this method was only illustrated for a single example, that is a modified generator with inertial nonlinearity,[22,23] we shall now present

results of its application using the Lorenz and Rössler models. We shall also report results obtained for this type of generator which were not completely reflected in our previous study.

METHOD

We shall analyze a certain generator of dynamic chaos whose mathematical model is known

$$\frac{d\mathbf{x}}{dt}=\mathbf{F}(\mathbf{x},\boldsymbol{\mu}^0),\quad \mathbf{x}\in R^n,\quad \boldsymbol{\mu}^0\in R^m, \tag{2}$$

where $\mathbf{x}$ is the vector of state, $\mathbf{F}$ is the vector of the right-hand sides of the model system, and $\boldsymbol{\mu}^0$ is the vector of the constant values of the parameters.

We shall implement relatively slow modulation of an arbitrary number of parameters of the information signals $\mu_i(t)$, i.e., we shall introduce the variables

$$\mu_i^*=\mu_i^0+\mu_i(t), \tag{3}$$

which allows several communications to be transmitted simultaneously. In this case, the signal transmitted by the communication channel, comprising a one-dimensional realization of the oscillatory process of the chaotic generator, is generated by a nonautonomous dynamic system

$$\frac{d\mathbf{x}}{dt}=\mathbf{F}(\mathbf{x},\boldsymbol{\mu}^0+\boldsymbol{\mu}(t)),$$
$$\boldsymbol{\mu}^0=(\mu_1^0,\mu_2^0,\ldots,\mu_m^0),$$
$$\boldsymbol{\mu}(t)=(\mu_1(t),\mu_2(t),\ldots,\mu_m(t)). \tag{4}$$

In order to solve the problem of synthesizing dynamic systems using an observable one-dimensional realization, the system (4) should be reduced to the form (1). This implies that by replacing the variables, we need to transform the right-hand sides such that instead of the vector function $\mathbf{F}$ there remains only a single scalar nonlinear function f, which may have a very complex form, such as

$$f=\frac{P}{Q},\quad P(\mathbf{x})=\sum_{l_1,l_2,\ldots,l_n=0}^{\nu_1} C_{l_1,l_2,\ldots,l_n}\prod_{k=1}^{n} x_k^{l_k},$$
$$\sum_{k=1}^{n} l_k\leqslant\nu_1,$$
$$Q(\mathbf{x})=\sum_{l_1,l_2,\ldots,l_n=0}^{\nu_2} D_{l_1,l_2,\ldots,l_n}\prod_{k=1}^{n} x_k^{l_k},$$
$$\sum_{k=1}^{n} l_k\leqslant\nu_2,\quad \nu_2<\nu_1, \tag{5}$$

where $C_{l_1,l_2,\ldots,l_n}$, $D_{l_1,l_2,\ldots,l_n}$ generally depends on time and is uniquely related to the parameters μ_i^* of the system (4).

For the model systems considered here, i.e., Lorenz, Rössler, and a generator with inertial nonlinearity, these transformations will be made in the following section. Since it is assumed that the average rate of change in the parameters is small compared with the base frequency of the generator oscillations (2), i.e., $\overline{d\mu_i^*/dt}\ll\overline{dx_j/dt}$ for any i and j, we can introduce the time interval t_0 during which the values of the parameters can permissibly be assumed to be almost constant, i.e., for times of the order of t_0 we can neglect the nonautonomy of the system (4). This means that it is possible to reconstruct instantaneous values of the system parameters using short sections of its one-dimensional realization, i.e., to reproduce the information signals $\mu_i(t)$ which parametrically modulate the chaotic generator.

If the condition $\overline{d\mu_i^*/dt}\ll\overline{dx_j/dt}$ is not satisfied, we must take into account the time derivatives of the parameters when transforming the system (4) to the form (1), which makes the nonlinear function f substantially more complex and impedes the solution of the global reconstruction problem.

By applying the reconstruction technique to the one-dimensional realization $x_1(t)$ of the chaotic generator which can be measured at the exit of the transmitting device, the information receiver, knowing the general form of the mathematical model (2), isolates the useful signals $\mu_i(t)$. To do this, the receiver must differentiate the realization $x_1(t)$ n times, thus determining the left-hand sides of the model system (1). As a result, the problem of determining the values of the parameters at a given time reduces to the need to solve an algebraic equation with a certain number of unknowns (when f can be represented in the form (5), these unknowns are the coefficients $C_{l_1,l_2,\ldots,l_n}$ and $D_{l_1,l_2,\ldots,l_n}$, which are uniquely related to the parameters μ_i^* of the system (4)). Since a discretized time dependence $x_1(i\Delta t)$ rather than an analog signal is required for computer processing, the derivatives are clearly determined at discrete times $i\Delta t$ using approximate numerical differentiation formulas.

We can approximate the unknown coefficients by writing a system of K algebraic equations ($K=[t_0/\Delta t]$) for L unknowns ($L\ll K$) and solving this by the least squares method. Quite clearly, the approximation error decreases with decreasing L. Thus, in order to implement the proposed method of secure communication in practice, the model system must be simplified as far as possible. If the series of parameters of the system (4) remains unchanged, it is advisable to assume that these are predefined and approximate for fewer unknowns.

MODELS INVESTIGATED

We selected the Lorenz, Rössler, and generator with inertial nonlinearity models as chaotic oscillators.

a) *Lorenz system*. We shall analyze the equations from the well-known Lorenz model

$$\frac{dx}{dt}=\sigma(y-x),\quad \frac{dy}{dt}=rx-y-xz,\quad \frac{dz}{dt}=-bz+xy. \tag{6}$$

We shall assume that the x coordinate of the system (6) is selected as the carrier signal. It was shown in Ref. 8 that for this case, the transformation of (6) to the form (1) can give a simpler form of the function f than that for the other two coordinates of the Lorenz equations. As a result of this transformation, which is made by the change of variables

$$X=x, \quad Y=\sigma(y-x), \quad Z=\sigma[(r+\sigma)x-(\sigma+1)y-xz], \tag{7}$$

we obtain the system

$$\frac{dX}{dt}=Y, \quad \frac{dY}{dt}=Z,\ldots, \quad \frac{dZ}{dt}=f(X,Y,Z,\boldsymbol{\mu}),$$

$$\boldsymbol{\mu}=(\sigma,r,b), \tag{8}$$

$$f=b\sigma(r-1)X-b(\sigma+1)Y-(b+\sigma+1)Z$$

$$-X^2Y-\sigma X^3+\frac{Y[(\sigma+1)Y+Z]}{X}, \tag{9}$$

which generally contains three unknown parameters. Using the Lorenz model as an example we demonstrate that secure transmission of information can be achieved by modulating only one of its parameters, for example b. In this case, formula (9) is best rewritten as follows:

$$f+(\sigma+1)Z+X^2Y+\sigma X^3-\frac{Y[(\sigma+1)Y+Z]}{X}$$

$$=b[\sigma(r-1)X-(\sigma+1)Y-Z]. \tag{10}$$

Since σ and r are assumed to be known, and Y, Z, and f can be determined by numerical differentiation of the time series $x(i\Delta t)$ obtained by integrating the system (6), where Δt is the discretization step, which is taken to be 0.025 for all the models studied, Eq. (10) is merely a linear algebraic equation with a single unknown.

Theoretically, in order to determine the instantaneous value of b, we need to know the phase coordinates and their derivatives only at one moment of time. In practice, we need to analyze a short section of the scalar time series and approximate the value of b using the results of calculations at different times in the interval t_0 during which the value of the parameter can be considered to be almost constant in order to improve the accuracy of calculating the parameter.

b) *Generator with inertial nonlinearity*. A modified generator with inertial nonlinearity[22,23] was taken as the second model system

$$\frac{dx}{dt}=m_0x+y-xz, \quad \frac{dy}{dt}=-x,$$

$$\frac{dz}{dt}=g_0z+0.5g_0(x+|x|)x. \tag{11}$$

We shall assume that the signal emitted by the generator is a one-dimensional realization $y(t)$. The system (11) is transformed to the form (1) by changing the variables

$$Y=y, \quad Z=-x, \quad X=-m_0x-y+xz, \tag{12}$$

as a result of which the generator equations have the form

$$\frac{dY}{dt}=Z, \quad \frac{dZ}{dt}=X, \quad \frac{dX}{dt}=f(X,Y,Z,\boldsymbol{\mu}), \quad \boldsymbol{\mu}=(m_0,g_0), \tag{13}$$

$$f=\frac{X(X+Y)}{Z}+(m_0g_0-1)Z$$

$$-g_0(X+Y)+0.5g_0(|Z|-Z)Z^2. \tag{14}$$

The model of a generator with inertial nonlinearity was used to illustrate the possibility of simultaneously transmitting two independent information signals along a single communication channel. For this purpose the parameter m_0 was modulated by a wide-band chaotic signal obtained by integrating the Rössler equation and the parameter g_0 was modulated by the harmonic signal

$$\frac{dx}{dt}=(m_0+dx_1)x+y-xz, \quad \frac{dy}{dt}=-x,$$

$$\frac{dz}{dt}=-g_0(1+k_1\sin(\omega t))[z-0.5(x+|x|)x],$$

$$\frac{dx_1}{dt}=k(-y_1-z_1), \quad \frac{dy_1}{dt}=k(x_1+ay_1),$$

$$\frac{dz_1}{dt}=k(b+z_1(x_1-c)), \tag{15}$$

where k is a constant which renormalizes the time in the Rössler model such that the process of variation of the parameter $m^*=m_0+dx_1$ is slower than the oscillations of the generator.

A transformation of the equation for a generator with inertial nonlinearity to the form (13) assuming slowly varying parameters can give the function f in the form (14) provided that m_0 and g_0 are replaced by $m^*=m_0+dx_1$ and $g^*=g_0(1+k_1\sin(\omega t))$, respectively, i.e.,

$$f-\frac{X(X+Y)}{Z}+Z=m^*g^*Z-g^*[X+Y+0.5(|Z|-Z)Z^2]. \tag{16}$$

If we introduce the notation $s^*=m^*g^*$, Eq. (16) can be considered to be a linear algebraic equation with two unknowns s^* and g^*, which are found using a short scalar time series by the least squares method. The instantaneous value of the parameter m^* can then be uniquely determined from a knowledge of s^* and g^*.

c) Rössler system.

We take the y coordinates of the Rössler model as the carrier signal

$$\frac{dx}{dt}=-y-z, \quad \frac{dy}{dt}=x+ay, \quad \frac{dz}{dt}=b+z(x-c). \tag{17}$$

By analogy with the previous model (11), we transform (17) to the form (13) by changing the variables[8]

$$Y=y, \quad Z=x+ay, \quad X=ax+(a^2-1)y-z. \tag{18}$$

We then obtain the following form of the nonlinear function f:

$$f=-b+(a-c)X-cY+(ac-1)Z$$

$$-aY^2-aZ^2-aXY+XZ+(a^2+1)YZ. \tag{19}$$

We assume that the vector $\boldsymbol{\mu}=(a,b,c)$ contains two unknown parameters b and c which are modulated by the information signals. Equation (19) can then be rewritten as follows:

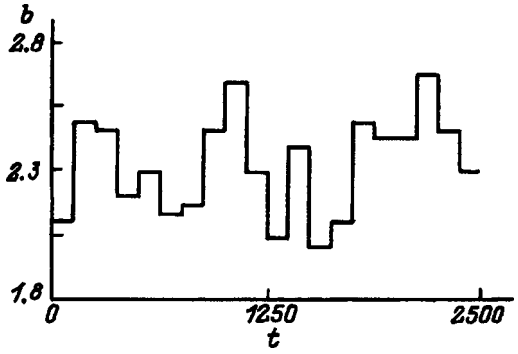

FIG. 1. Law of variation of the parameter b in a Lorenz system.

$$f-aX+Z+aY^2+aZ^2+aXY-XZ-(a^2+1)YZ = -b-c(X+Y-aZ). \quad (20)$$

The unknowns b and c of the linear algebraic equation (20) can again be obtained by applying the least squares method to the results of calculating the parameters at the times $i\Delta t$ of the discretized signal $y_i=y(i\Delta t)$ within the short time t_o. The Rössler model is used to illustrate the possibility of transmitting graphical information by modulating two of its parameters with the information signals.

RESULTS

Before giving some specific results which confirm the efficiency of the proposed method of secure communications, we shall make a few observations. A characteristic feature of the global reconstruction method is that it can be applied not only to steady-state signals but also to transient processes. Although the Takens theorem,[2] to which reference is usually made when reconstructing phase portraits, was demonstrated for the case where the signal is a one-dimensional projection of a phase trajectory assigned to the attractor of a dynamic system, the assignment of the phase trajectory to the attractor is not a necessary condition for the modeling problem. We have already observed that because of the relatively slow variation of the parameters, we can introduce the time interval t_0 during which it is permissible to assume that the values of μ_i^* in Eq. (3) are almost constant and the system (4) is autonomous. Since for the same values of the parameters the motion along the attractor and the transient processes are described by the same equations, the transience of the signal in the communications channel during the time t_0 is not of fundamental importance for determining the instantaneous values of μ_i^*.

We also note that any method of transferring information should be analyzed in terms of its performance under noise conditions of varying origin. Thus, in all the examples examined below, a normally distributed random quantity with a variance of 10^{-4} was added to the information signals performing the parametric modulation.

a) *Lorenz system.* We shall take the law of variation of the parameter b to be a stepped time dependence similar to that plotted in Fig. 1. This type of signal can be used to transmit graphical information. To illustrate this we shall scan the well-known Raphael painting of the Sistine Madonna (Fig. 2a) with 200×300 resolution. We shall divide the range of variation of the parameter $b\in[2-3]$ into 256 subranges, each corresponding to a shading gradation of the black and white image. The useful signal is a dependence such as that shown in Fig. 1, where each subrange of variation of the parameter corresponds to a particular step height. The carrier signal (the time dependence $x(t)$ of the Lorenz system), divided into the same number of subranges as the information signal, is shown in Fig. 2b. By applying the global reconstruction method, we isolated the modulation signal (Fig. 2c). Similar results were obtained by modulating the parameter r in system (6).

We note that having selected a stepped time dependence

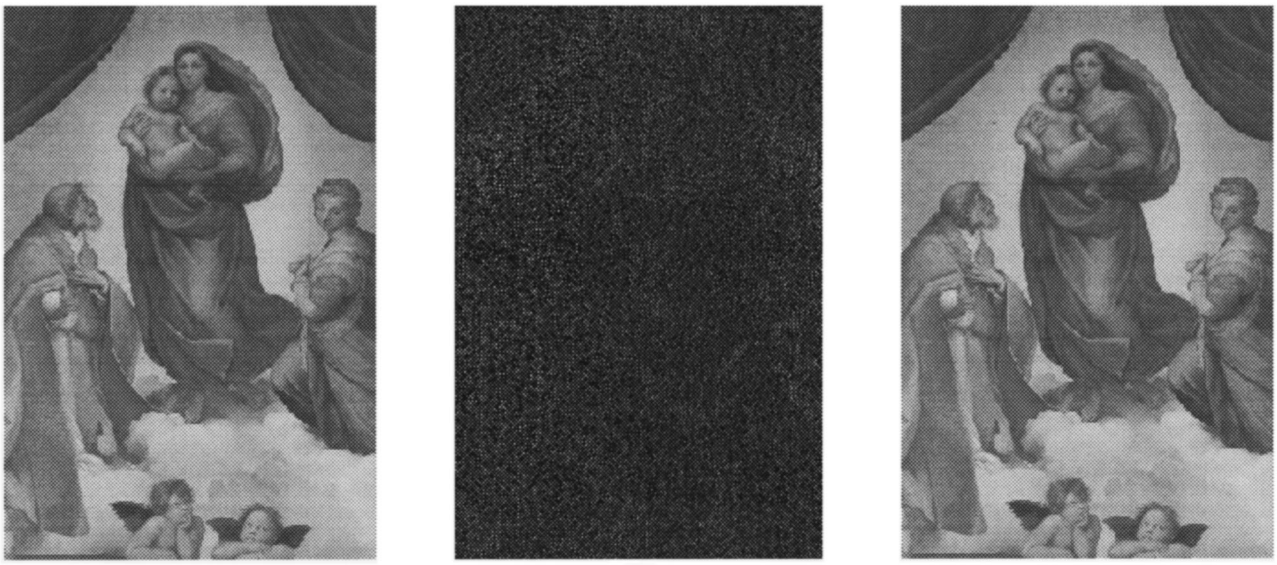

FIG. 2.

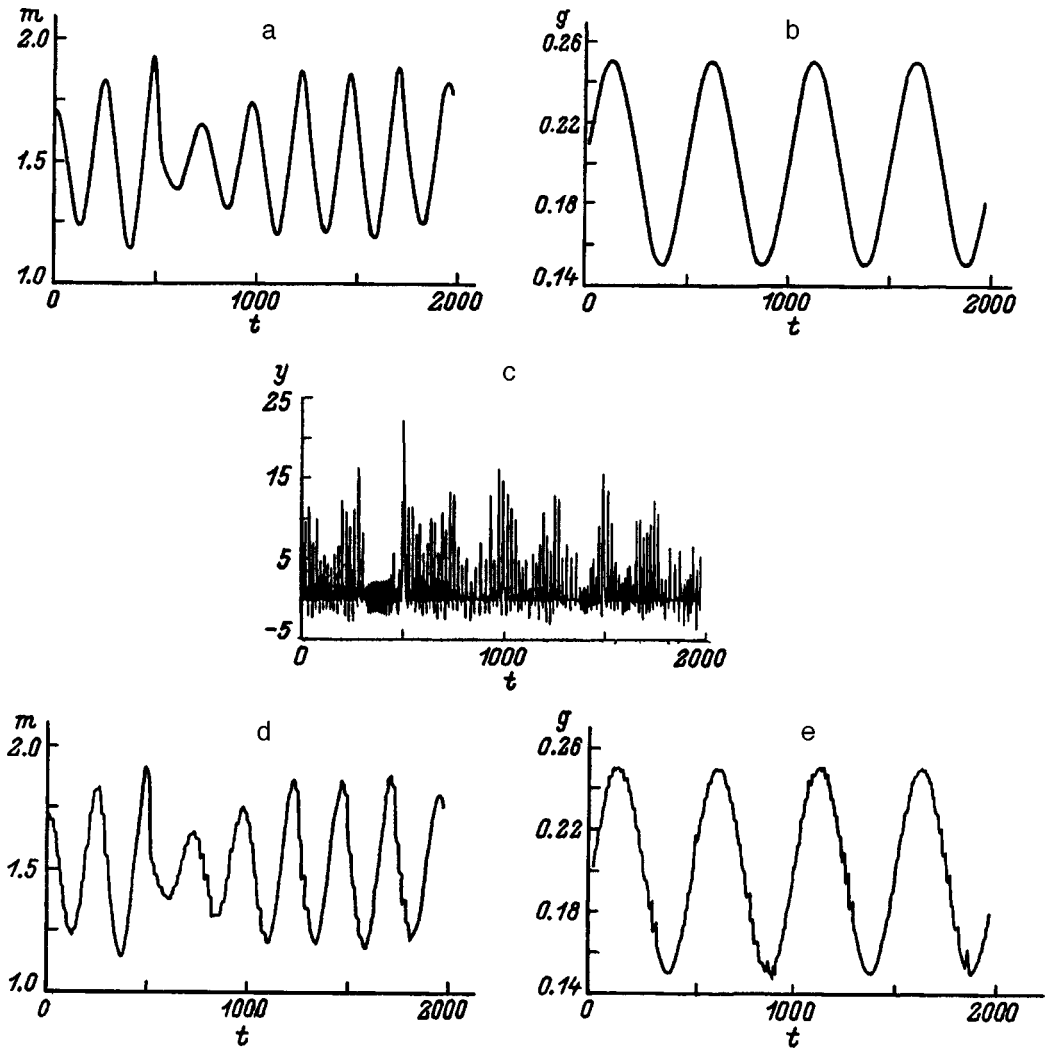

FIG. 3.

as the law of parametric modulation, we must recognize that we cannot use formula (10) for the switching time between subranges since it is abundantly clear that in the immediate vicinity of the switching time the system parameter cannot be considered to be slowly varying. Thus, the derivatives $d\mu_i/dt$ must be taken into account when transforming the system (4) to the form (1). This can be avoided by selecting fairly large widths for each step and disregarding the small sections near the switching times.

b) *Generator with inertial nonlinearity.* We shall demonstrate the possibility of transmitting two signals simultaneously: a chaotic and a regular one. We turn our attention to the system (15) in which we select the following values for the constants: $d=0.025$, $g_0=0.2$, $m_0=1.5$, $\omega=0.006$, $k_1=0.05$, $k=0.025$, $a=0.15$, $b=0.2$, and $c=10.0$. The time dependences of the parameters m^* and g^* of this system are illustrated in Figs. 3a and 3b. The information receiver, knowing the form of the nonlinear function (16), receives the signal (Fig. 3c) and reconstructs the modulation signals (Figs. 3d and 3e).

c) *Rössler system.* By analogy with the Lorenz model, we take a stepped time dependence as the parametric modulation law which allows us to transmit graphical information. However, two of its parameters, b and c, will now be modulated by the useful signals. We take two fragments of Leonardo da Vinci's painting ''Madonna of the Rocks'' (Figs. 4a and 4b) and scan them with 200×250 resolution. The range of variation of each parameter $b\in[0.1-0.3]$ and $c\in[8-12]$ is again divided into 256 subranges. Figure 4 shows the initial fragments of the painting (Figs. 4a and 4b) reconstructed using the global reconstruction technique (Figs. 4d and 4e), and also the signal in the communication channel (Fig. 4c).

A disadvantage of this method of transmitting a graphical image is that the information receiver must know the resolution which was used for scanning since the isolated signal is a single realization. This can be circumvented as follows. Let us assume that one of the parameters is modulated by a signal carrying information on the scanned graphical image. Simultaneous modulation of a second parameter can transmit information on the resolution. If we introduce the x and y coordinates of points on the transmitted fragment of the painting, one method of transmitting the graphical image is shown in Fig. 5. The number of steps (Fig. 5b) will carry information on the resolution along the y axis while the resolution along the x axis can be determined from the ratio

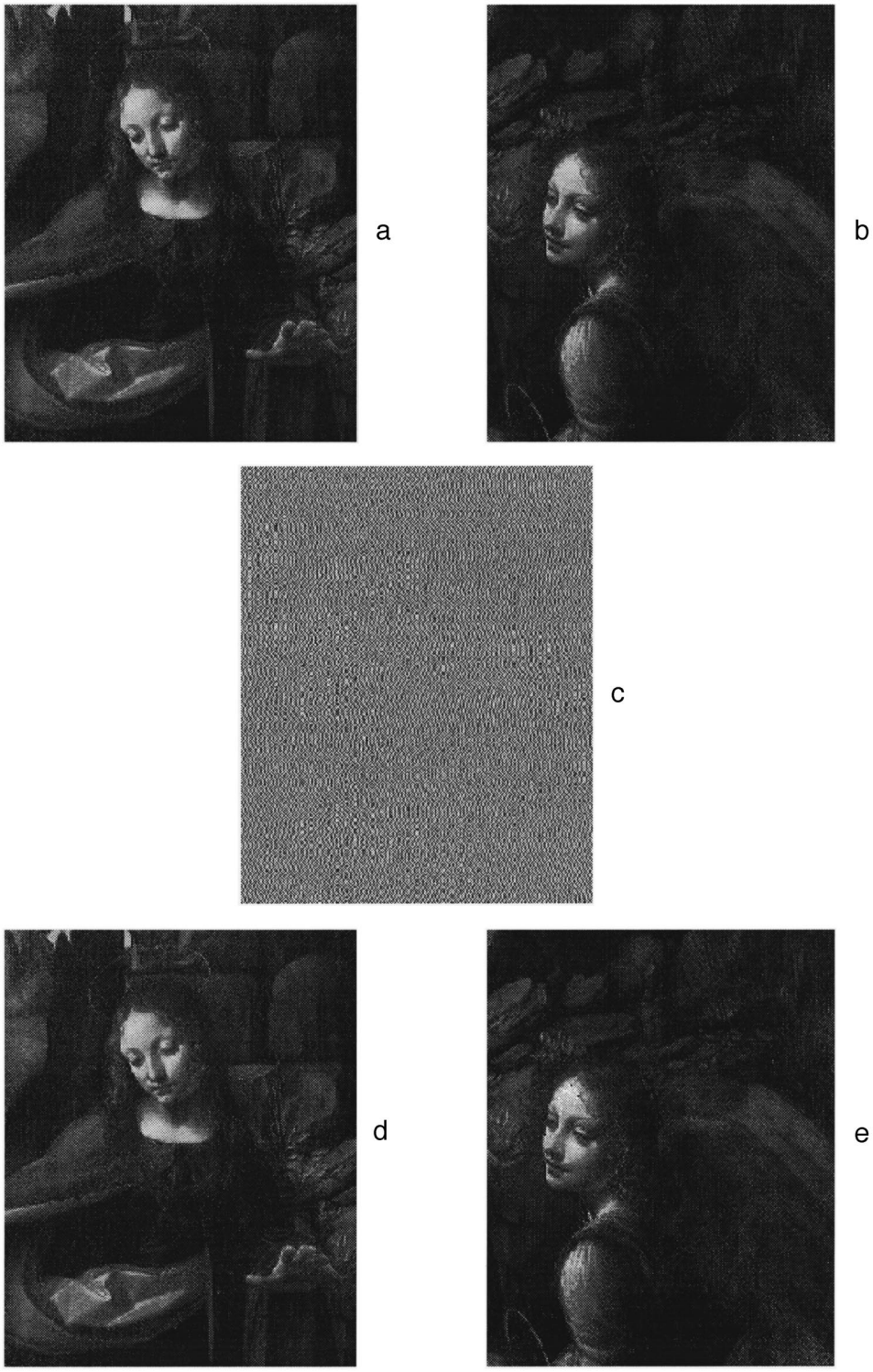

FIG. 4.

of the step width to the corresponding value for Fig. 5a.

The possibility of simultaneously modulating different parameters of a dynamic system by information signals means that the methods of transmitting a graphical image can be varied. In the simplest case when some drawing is transmitted using only two colors (black and white), i.e., the useful signal is a binary sequence of symbols (0 or 1), the rate of transmission of the information by modulating two parameters of the dynamic system can be enhanced by transmitting only the coordinates of points corresponding to one particular color. Quite clearly, other methods of transmitting graphical information can also be suggested.

CONCLUSIONS

Here we have illustrated a new method for secure communication based on the global reconstruction of dynamic

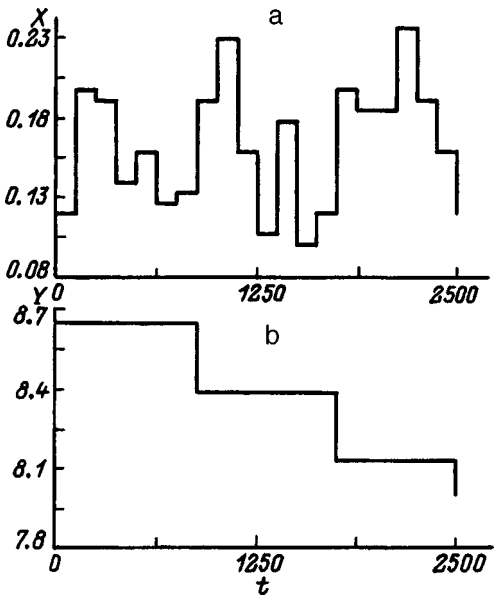

FIG. 5. One method of transmitting graphical information by simultaneously modulating two parameters.

systems. The selected model systems were Lorenz and Rössler systems and a generator with inertial nonlinearity. The results confirm the efficiency of the proposed method and also its reliability in the presence of noise considerably exceeding the thermal and shot noise of real electronic devices.

Theoretically, the proposed method imposes no constraints on the number of parameters which can be varied simultaneously. In practice such constraints arise from the finite accuracy of the calculations of μ_i^*. In particular, the instantaneous values of μ_i^* were not determined with a high degree of accuracy when three parameters were modulated simultaneously for the same discretization step ($\Delta t = 0.025$) and 256 subranges. Thus, when more than two parameters are modulated, fewer subranges must be used or the step Δt must be reduced, thereby increasing the number of carrier signal points contained in the range t_0. In this last case, the parameters of the numerical system must be optimized and these may differ for different model systems.

These constraints can be ascribed to the purely technical aspect of implementing the secure transmission of information, on which we decided not to focus attention in the present study, confining ourselves to demonstrating the fundamental possibility of simultaneously transmitting several information signals independently in a single communication channel using the global reconstruction technique.

This work was partly financed by the Russian State Committee on Higher Education (Grant No. 95-0-8.3-66).

[1] N. H. Packard, J. P. Crutchfield, J. D. Farmer, and R. S. Shaw, Phys. Rev. Lett. **45**, 712 (1980).
[2] F. Takens, in *Dynamical Systems and Turbulence*, edited by D. A. Rang and L. S. Young, Lecture Notes in Mathematics, Vol. 898 (Springer-Verlag, Berlin, 1981), p. 366.
[3] J. D. Farmer and J. J. Sidorowich, Phys. Rev. Lett. **59**, 845 (1987).
[4] M. Casdagli, Physica D **35**, 335 (1989).
[5] P. Grassberger and J. Procaccia, Phys. Rev. Lett. **50**, 346 (1983).
[6] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, Physica D **16**, 285 (1985).
[7] J. Cremers and A. Hübler, Z. Naturforsch., A: Phys. Sci. **42**, 797 (1987).
[8] G. Gouesbet and C. Letellier, Phys. Rev. E **49**, 4955 (1994).
[9] J. Kadtke and M. Kremliovsky, AIP Conf. Proc. **375**, 189 (1995).
[10] O. L. Anosov, O. Ya. Butkovskii, Yu. A. Kravtsov, and E. D. Surovyatkina, AIP Conf. Proc. **375**, 71 (1995).
[11] J. L. Breeden and N. H. Packard, Int. J. Bifurcation Chaos Appl. Sci. Eng. **4**, 311 (1994).
[12] V. S. Anishshenko and A. N. Pavlov, Phys. Rev. Lett. (in press).
[13] K. M. Coumo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).
[14] K. M. Kocarev, K. S. Halle, K. Eckert *et al.*, Int. J. Bifurcation Chaos Appl. Sci. Eng. **2**, 709 (1992).
[15] C. W. Wu and L. O. Chua, Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 1619 (1992).
[16] U. Parlitz, L. O. Chua, L. Kocarev *et al.*, Int. J. Bifurcation Chaos Appl. Sci. Eng. **2**, 973 (1992).
[17] K. M. Coumo, A. V. Oppenheim, and S. H. Strogatz, IEEE Trans. Circuits Syst. **40**, 626 (1993).
[18] H. Dedieu, M. P. Kennedy, and M. Hasler, IEEE Trans. Circuits Syst. **40**, 634 (1993).
[19] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[20] S. Hayes, C. Grebogi, E. Ott, and A. Mark, Phys. Rev. Lett. **3**, 1781 (1993).
[21] H. D. I. Abarbanel and P. S. Linsay, IEEE Trans. Circuits Syst. **40**, 643 (1993).
[22] V. S. Anishchenko, *Complex Oscillations in Simple Systems* [in Russian], Nauka, Moscow (1990), 312 pp.
[23] V. S. Anishshenko, *Dynamical Chaos — Models and Experiments* (World Scientific, Singapore, 1995), p. 383.

Translated by R. M. Durham