COMPUTING LYAPUNOV EXPONENTS FROM RR-INTERVALS

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Abstract— We discuss the possibility to compute dynamical properties of chaotic attractor from threshold-crossing interspike intervals and potential application of dynamical characteristics as a possible diagnostic tool in medicine and biology.

I. INTRODUCTION

The traditional approach to obtaining of experimental data consists in choosing of the sampling step Δt and storing the values of some state variable $s(t)$ corresponding to the time moments $i\Delta t$. But, there is also another way of getting experimental data, namely, to generate spike-train data [1]. There are some situations (being typical in biology) when the time intervals alone contain necessary information about the system under study and a full realization of a process $s(t)$ may be redundant. For example, modern cardiology says that RR-intervals of electrocardiogram contain all the necessary information about the state of cardio-vascular system of organism in general when P,Q,R,S,T-waves arise return with small distortions from beat to beat. There are similar notions in neuro dynamics where in practice one does not process signals generated by neurons. Instead of this, the time intervals between successive spikes are analyzed, i.e., interspike intervals (ISI).

The aim of the present paper is to investigate the possibility to restore dynamical properties of chaotic attractor from ISI and to exploit dynamical characteristics as possible diagnostic tools in cardiology.

II. RECONSTRUCTION OF DYNAMICS FROM INTERSPIKE INTERVALS

There are two general methods for recording spike-train data. Some condition is imposed to the signal $s(t)$ in both cases. If this condition is fulfilled a spike is generated and intervals between two successive spikes are stored.

1-st method. Some threshold level θ is chosen and interspike intervals T_i are measured between successive crossings by $s(t)$ of this level in one direction.

2-nd method. The sequences of time intervals T_i are generated recursively using integrate-and-fire model (or some other neuron model [2]):

$$\int_{t_i}^{t_{i+1}} s(t)dt = \theta, \quad T_i = t_{i+1} - t_i. \tag{1}$$

The works [3] establish the possibility to completely reconstruct the original attractor using the latter method for obtaining spike-train data. Up to now it seemed to be impossible to reconstruct the original attractor using threshold-crossing interspike intervals alone [3].

In [4] we have demonstrated the possibility to reconstruct metrical and dynamical properties of chaotic attractors using only the sequence of threshold-crossing interspike intervals T_i or the time intervals between the successive maxima (or minima) of initial time series. The concept of instantaneous frequency has been considered to reveal the meaning of threshold-crossing ISI.

According to Takens theorem [5] a smooth realization is required to solve the reconstruction problem. This realization should be a one-dimensional projection of the phase trajectory of attractor. To pass from the discrete signal to the continuous one it is necessary to introduce the phase for the sequence of interspike intervals. At present three ways of how to introduce the phase of chaotic oscillations are known [6].

The first two ways are connected with the existence of a projection of system's attractor on the plane (x, y) reminding a smeared limit cycle. If this projection exists then one is able to introduce the Poincarè secant so that it would pass through an equilibrium point of the system around which the motion occurs. According to the *first definition*, the phase is defined via the time moments of trajectory's crossings of the secant surface t_i as follows:

$$\varphi^i(t) = 2\pi\frac{t - t_i}{t_{i+1} - t_i} + 2\pi i, \quad t_i \le t < t_{i+1}. \tag{2}$$

The *second definition* operates with the above mentioned projection, and the phase is introduced as

$$\varphi^p = \arctan(y/x). \quad (3)$$

The *third definition* is based on the analytic signal concept $z(t) = s(t)+js^H(t)$, where s^H is Hilbert transform of signal $s(t)$:

$$s^H(t) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau. \quad (4)$$

P.V. means that the integral is taken in the sense of Cauchy principal value. Instantaneous amplitude $A(t)$ and phase $\varphi^H(t)$ of $s(t)$ are uniquely defined from $z(t)$, as well as instantaneous frequency being the derivative of $\varphi^H(t)$.

In [4] we have discussed the existence of correlations between the points $\omega^i(t_i) = 2\pi/T_i$ and the points of averaged instantaneous frequency $<\omega^H>(t_i) = \frac{1}{T_i}\int_{t_i}^{t_{i+1}} \omega^H(t)dt$. We have connected the points of $\omega^i(t_i)$ by a smooth curve to get an idea of how the real frequency averaged over a moving window with varying length would behave, at least qualitatively (Fig. 1). Our approach allowed us to reconstruct dynamical properties of chaotic attractor with good precision (Fig. 2).

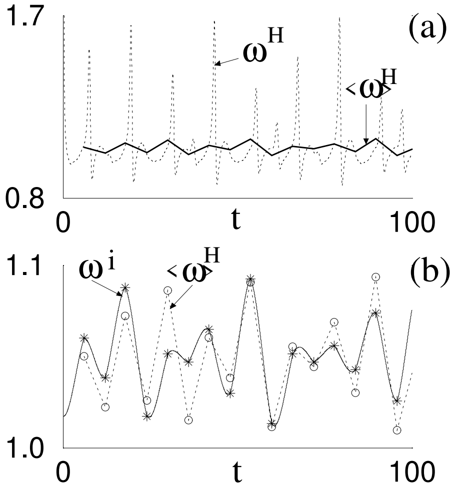

Figure 1: (a) - Time dependence of instantaneous frequency ω^H obtained from the 1-st coordinate of Rössler system using the Hilbert transform; (b) - the points $\omega^i(t_i)$ connected by solid curve and the values $<\omega^H>(t_i)$ connected by dashed line.

Note, that the passing from $s(t)$ to $\omega^i(t)$ means passing to slow time dependence what may lead to some problems connected with processing of realizations $s(t)$ including not very many basic periods of oscillations. In this case we suggest to use simpler method for phase introduction (2), which has a worse precision comparing to the *third definition* of phase but does not change the temporal scale of the signal under study.

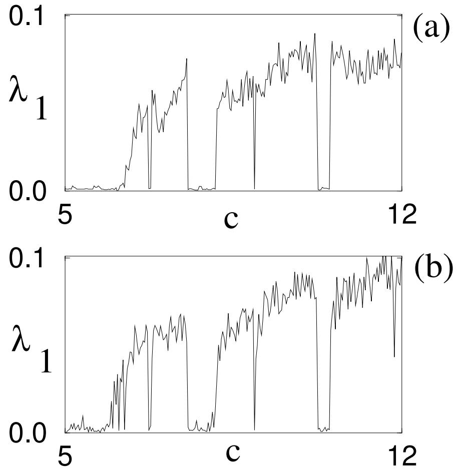

Figure 2: (a) - Largest Lyapunov exponent calculated from x coordinate of the system (7) vs c; (b) - largest Lyapunov exponent calculated from $\omega^i(t)$ of the system (7) vs c

Basing on the formula (2) we suggest to transform the sequence of ISI into a smooth temporal dependence:

$$a(t) = \cos 2\pi \frac{t-t_i}{t_{i+1}-t_i}, \quad t_i \le t < t_{i+1}. \quad (5)$$

Since the growth of phase in the interval between the two spikes is considered to be linear according to the definition (2) and the segments of cosines with different periods are "sewed together" at the points $\varphi = \pm 2\pi i$ the first derivative of the signal (5) does not suffer discontinuities. Thus, the suggested way of transition from a discrete series to a smooth signal (5) allows one to apply the method of largest Lyapunov exponent computation [7] to $a(t)$.

Consider now how the suggested "cosine method" works for test models, namely, for modified oscillator with inertial nonlinearity (Anishchenko-Astakhov oscillator [8]):

$$\frac{dx}{dt} = mx+y-xz, \frac{dy}{dt} = -x, \frac{dz}{dt} = -gz+0.5g(x+|x|)x \quad (6)$$

and Rössler system [9]:

$$\frac{dx}{dt} = -(y+z), \frac{dy}{dt} = x+ay, \frac{dz}{dt} = b+z(x-c). \quad (7)$$

Figs. 3 and 4 allow one to get sure that even if the largest LCE computed from the signal $a(t)$ does not coincide with the true value of the exponent (Fig. 4) it follows qualitatively the system's dynamics. Since when solving the diagnostics problem we are interested not in obtaining the exact value of Lyapunov exponent, but in establishing some objective criteria of different dynamical behavior, it is not of principal importance whether we compute the true value of LCE or some quantity which is proportional to it. Let us now turn to experimental results.

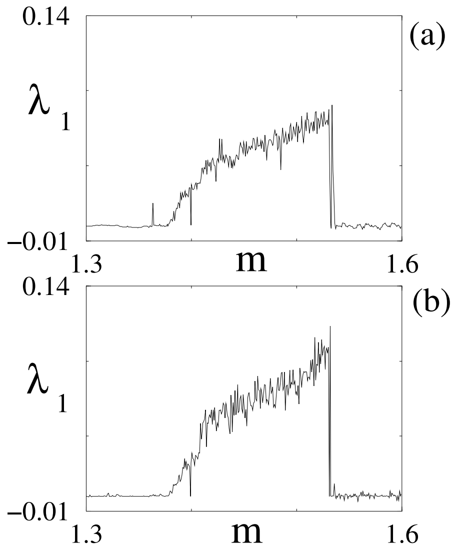

Figure 3: Dependence of the value of Lyapunov characteristic exponent on the values of control parameter of oscillator with inertial nonlinearity: (a) is the true values of exponents for the systems (6); (b) is the Lyapunov exponents computed from the signal (5) when the time intervals between the successive maxima of coordinate $x(t)$ were used as an analogue of RR-intervals.

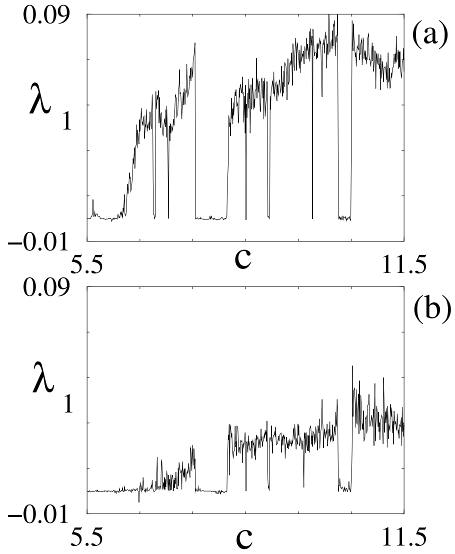

Figure 4: Dependence of the value of Lyapunov characteristic exponent on the values of control parameter of Rössler system: (a) is the true values of exponents for the systems (7); (b) is the Lyapunov exponents computed from the signal (5).

III. APPLICATION OF THE DYNAMICAL CHARACTERISTICS TO DIAGNOSTIC IN CARDIOLOGY

Recently, there has been a growth of interest of specialists in nonlinear dynamics to the problems of diagnostics in medicine and biology [10]. Signals of electric activity of heart, namely, electrocardiograms (ECGs), attract the attention of many researchers. For the present moment a large amount of works is known devoted to the study of ECG which has been fulfilled by either specialists in medicine with the use of some knowledge of nonlinear dynamics or by the physicists not possessing fundamental knowledge in physiology but having a large experience in processing the signals of complex structure (e.g., [10]). A large number of publications in this area require some generalization (standardization) of the known methods for time series analysis to reveal the most informative ones which can be recommended for diagnostic [11].

To compare different methods for time series analysis from the viewpoint of their effectiveness as diagnostic tools for a selected concrete example we consider the following problem. We use a data banks of RR-intervals of patients after myocardial infarction (29 persons). Those patients who survived after infarction formed the first group (14 people) while those who has died during a month as a consequence of infarction were included into the second group (15 people).

The problem was formulated as follows: "Is it possible to find objective differences in the sequences of cardiointervals of given patients which have been recorded during the first days after myocardial infarction to make a prognosis of their survival from RR-sequences?"

At least 10 different characteristics were computed such as average value ($\overline{RR}$), variance (σ^2_{RR}), energy E_{LF} in the range of lower frequencies [0.04, 0.15] Hz, energy E_{HF} in the range of higher frequencies [0.15, 0.4] Hz, the ratio of energies (E_{LF}/E_{HF}), the normalized entropy $H_p = H/E$, where H is Shannon entropy ($H = -\sum_{i=1}^{N} p_i \ln p_i$) and E is a magnitude proportional to the total energy of the system in the frequency range meaningful for diagnostic, an entropy computed on the power spectrum and normalized to the value $E - H_s$ [12], largest Lyapunov exponent computed by means of the algorithm [7] for the "cosine method" (λ_1), etc.

All of these characteristics were computed for RR-sequences recorded on the 1-st day after infarction (the measurement time was ≤ 5 minutes). The best results were obtained for Lyapunov exponent. According to our computation, for 11 patients of 14 from the first group the value of λ_1 falls into the interval $[0.5 \div 1.1]$

bit/sec, while for 11 patients from the second group λ_1 was outside this interval.

Other characteristics give worse results (see, for example, a table below)

Number of group	Number of patients	Diagnostic using			
		$\overline{RR}$	σ^2_{RR}	H_s	λ_1
1	14	9	9	8	11
2	15	10	11	10	11

IV. CONCLUSIONS

Our research includes the data measured from only 29 patients. Certainly, larger groups are necessary to verify the possibility for diagnostics according to the value of λ_1. We would like to note, that λ_1 was unable to distinguish between two groups using the RR-sequences measured several days after myocardial infarction for the same patients. But, we suppose that in any case the largest Lyapunov exponent may be considered as one of the potential diagnostic tools.

The advantage of the "cosine method" being discussed in the present work is that it can be applied not only to RR-intervals, but also to the series of other data, for example, to interspike intervals for neurons.

The transition from the sequence of discrete values to the smooth time dependence and the possibility to compute dynamical characteristics for the signals generated by neurons allow us to hope that Lyapunov exponents will be added to the number of characteristics which are traditionally computed in recent years for the data of ISI type.

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References

[1] A. Longtin, *Int. J. Bif. Chaos*, **3**, 651 (1993); A. Longtin, A. Bulsara, and F. Moss, *Phys. Rev. Lett.*, **67**, 656 (1991); H. Tuckwell, Introduction to Theoretical Neurobiology (Cambridge University Press, Cambridge, 1988), Vols. 1 and 2; A. Garfinkel, M.L. Spano, W.L. Ditto, and J.W. Weiss, *Science* **257**, 1230 (1992); S.J. Schiff, K. Jerger, D.H. Duong, T. Chang, M.L. Spano, and W.L. Ditto, *Nature*, **370**, 615 (1994); D. Pierson, and F. Moss, *Phys. Rev. Lett.*, **75**, 2124 (1995); D. Christini, and J.J. Collins, *Phys. Rev. Lett.*, **75**, 2782 (1995); X. Pei, and F. Moss, *Nature*, **379**, 618 (1996); P. So, E. Ott, S. Schiff, D. Kaplan, T. Sauer, and C. Grebogi, *Phys. Rev. Lett.*, **76**, 4705 (1996).

[2] D.M. Racicot, and A. Longtin, *Physica D*, **104**, 184 (1997).

[3] T. Sauer, *Phys. Rev. Lett.*, **72**, 3911 (1994); R. Castro, and T. Sauer, *Phys. Rev. Lett.*, **79**, 1030 (1997).

[4] N.B. Janson, A.N. Pavlov, A.B. Neiman, and V.S. Anishchenko, *Phys. Rev. E*, (accepted for publication for vol.58, issue 1, 1998).

[5] F.Takens, in *Dynamical Systems and Turbulence* (Warwick, 1980), ed. by D.A. Rang and L.S. Young, Lecture Notes in Mathematics Vol. 898 (Springer-Verlag, Berlin, 1981), p. 366.

[6] A.S. Pikovsky, M.G. Rosenblum, G.V. Osipov, and J. Kurths, *Physica D*, **104**, 219 (1997).

[7] A. Wolf, J.B. Swift, H.L. Swinney, and J.A. Vastano, *Physica D*, **16**, 285 (1985).

[8] V.S. Anishchenko, V.V. Astakhov, in *Nonlinear Dynamics In Circuits*, ed. by T.Carroll and L.Pecora, (World Scientific, 1995), p. 55; V.S. Anishchenko, "Dynamical Chaos – Models and Experiments," ed. by L. Chua, World Scientific, Singapore. 400 p., 1995.

[9] O.E. Rössler, *Phys. Lett. A*, **57**, 397 (1976).

[10] A.L. Goldberger, B.J. West, *Yale J. Biol. Med.*, **60**, 421 (1987); L. Glass, M.C. Mackey, *From Clocks to Chaos: The Rhythms of Life* (Princeton Univ. Press, Princeton, 1988); L. Glass, *Theory of heart* (New York. Springer-Verlag, 1991) C.K. Peng, J. Mietus, J.M. Hausdorff, S. Havlin, H.E. Stanley, and A.L. Goldberger, *Phys. Rev. Lett.*, **70**, 1343 (1993); C.K. Peng, S. Havlin, H.E. Stanley, and A.L. Goldberger, *CHAOS*, **5**, 82 (1995); A. Babloyantz, and A. Deslexhe, *Biol. Cybernetics*, **58**, 203 (1988); A. Voss, J. Kurths, H.J. Kleiner, A. Witt, P. Saparin, R. Dietz, H. Fiehring, and N. Wessel, *Biomedizinische Technik*, **39** 134 (1994); R.J. Storella, Y. Shi, H.W. Wood, M.A. Jimenez-Montano, A.M. Albano, and P.E. Rapp, *Int. J. Bifurcation Chaos*, **6**, 2169 (1996); J.J. Zebrowski, W. Poplawska, and R. Baranowski, *Acta Physica Polonika B*, **26**, 1055 (1995).

[11] Task Force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology, *Circulation*, **93**, 1043 (1996).

[12] Yu. L. Klimontovich, " Turbulent Motion and the Structure of Chaos", (Kluwer-Academic, Dordrecht, 1991); J. Kurths, A. Voss, P. Saparin, A. Witt, H.J. Kleiner, N. Wessel, *CHAOS*, **5**, 88 (1995).