# MULTICHANNEL COMMUNICATION BY MEANS OF GLOBAL RECONSTRUCTION

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***Abstract***— We restore the temporal dependences of control parameters of chaotic system from one-dimension realization. A possible solution of the problem of multichannel private communication is proposed using the presented technique.

Recently, there has been much interest in application of chaotic systems to communication. The known methods for private communication [1-6] are based as a rule on exploiting broadband aperiodic oscillations of some dynamical chaos oscillator as a carrying (or masking) signal. Information message being usually extracted from the chaotic carrier using the phenomenon of chaotic synchronization. In the pioneering works on this research problem there were two possible solutions suggested.

According to the first one [1,2], information signal $c(t)$ is added to the chaotic oscillations $h(t)$ of oscillator $G_1$ at the output of the transmitter, i.e., a signal in a channel of communication consists of two terms: $s(t) = h(t) + c(t)$, the power of signal $c(t)$ being much less than the power of $h(t)$. In the receiver a synchronous chaotic response $h^*(t) = h(t)$ is excited being then subtracted from $s(t)$.

For the second way [3-5] one needs to have also identical chaotic oscillators in the transmitter ($G_1$) and the receiver ($G_2$). The control parameter of $G_1$ is modulated by binary code. The excitation of synchronous regime (the corresponding parameters of $G_1$ and $G_2$ coincide) or the absence of synchronization (the parameters do not coincide) gives the possibility to restore by the recipient of information the initial sequence of binary symbols ("0" or "1").

A problem of simultaneous transfer of several messages in a single carrier has been studied in [7,8]. The suggested method supposed modulation of several control parameters of chaotic oscillator $G_1$ by information signals, the latter being extracted from the chaotic background by means of *autosynchronization* phenomenon.

We have suggested [9,10] an alternative method for multichannel private communication using the technique of global reconstruction. This technique is widely used as one of the methods for mathematical modeling from experimental data and has possible application to the problem of long-term prediction.

Global reconstruction [11-13] allows one to obtain the mathematical description in the form

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x},\ \vec{\mu^0}); \qquad \vec{x} \in R^n,\ \ \vec{\mu^0} \in R^m \qquad (1)$$

from an observable $x_1(t)$, the latter being realized in 2 stages. First of all, the embedding dimension $n$ is chosen and the phase portrait of dynamical system is restored [14,15]. At the second stage the general form of mathematical model (1) is stated (the form of nonlinear vector-function $\vec{F}(\vec{x}, \vec{\mu^0})$ is indicated *a priori*) and evolution equations are concretized, i.e., the value of parameter vector $\vec{\mu^0}$ is found.

Since there is no possibility to indicate even an approximate form of nonlinearities (the latter being a consequence of ill-posing of the inverse problem) the functions $F_j(\vec{x},\ \vec{\mu^0})$ are presented as a polynomial expansion with a finite number of terms, the expansion coefficients being fitted numerically using least squares algorithm. An arbitrariness in the choice of the dynamical description is a serious shortcoming of the method of global reconstruction and the obtained mathematical models need to be carefully treated.

Suppose, that the explicit form of nonlinear function $\vec{F}(\vec{x}, \vec{\mu^0})$ is known and only the parameters have to be found. Such a supposition may be treated as a particular case of the global reconstruction, but it may be effectively applied in a series of practical tasks. One of them is the problem of multichannel private communication, whose possible solution is discussed in the present paper.

The suggested method is rather simple. By analogy with [7,8] information signals perform modulation of several parameters of a dynamical system, but we shall use another way for demodulation. The idea of our method consists in the following. Let a dynamical chaos oscillator, whose mathematical model is known (i.e., the dynamics of the oscillator is described by the known nonlinear system (1) with $n$ phase variables and $m$ constant parameters) be a transmitter. We shall restrict ourselves to the class of DS's, whose equations permit transformations into the form

$$\frac{dx_1}{dt} = x_2, \quad \frac{dx_2}{dt} = x_3, \quad \ldots, \quad \frac{dx_n}{dt} = f(\vec{x},\ \vec{\mu^0}). \quad (2)$$

It is indeed possible to realize this transformation for many well-known models such as Lorenz, Rössler systems, Chua's circuit, etc. (see, e.g., [13]).

In radiophysical tasks of information transmission the modulating signals are low-frequency in comparison to the carrier. Using this circumstance, we consider relatively slow modulation of parameters

$$\mid \overline{\frac{d\mu_i}{dt}} \mid \ << \ \mid \overline{\frac{dx_j}{dt}} \mid \quad (3)$$

to transmit $m$ messages simultaneously

$$\mu_i^* = \mu_i^0 + \mu_i(t), \qquad i = 1, \ldots, m, \quad (4)$$

where $\mu_i^0$ are the constant values of control parameters of oscillator (1) and $\mu_i(t)$ are information signals. The signal transmitted via the communication channel is now described by the nonautonomous system

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x},\ \vec{\mu^*}(t)). \quad (5)$$

Let the recipient know the form of nonlinearities $\vec{F}_j(\vec{x}, \vec{\mu^0})$ except the values of concrete parameters $\vec{\mu^0}$. Then he is able to uniquely solve the problem of reconstruction of the system (5) from one-dimensional realization $x_1(t)$. The solution of the reconstruction problem means the extraction of information signals $\mu_i(t)$.

The condition (3) allows one to choose the temporal window of size $t^*$, inside which the values of $\mu_i^*$ are practically constant. The latter means, that nonautonomous properties of system (6) may be not taken into account during $t^*$. Shifting the temporal window along the carrier signal $x_1(t)$ one may extract the modulation signals $\mu_i(t)$ in real time. The computation of the current values of $\mu_i(t)$ is realized in practice as follows. The carrier $x_1(t)$ is differentiated $n$ times to compute the temporal dependencies $x_j(t),\ j = 2, \ldots, n$ and the left-hand sides of system (2). All the derivatives are computed at discrete time moments $i\Delta t$ using approximate formulas for numerical differentiation. To define the parameters $\mu_i^*$ at each time moment one should solve the following equation

$$\frac{dx_{n,i}}{dt} = f(\vec{x}_i,\ \vec{\mu}^*), \quad (6)$$

where $x_{n,i} = x_n(i\Delta t)$, $\vec{x_i} = \vec{x}(i\Delta t)$. This is an algebraic equation since the left-hand side of (6) is computed numerically and only vector $\vec{\mu^*}$ has to be found. We may write Eq. (6) for $i = 1, \ldots, K$, $K = [t^*/\Delta t]$ and obtain a system of K equations with $m$ unknown variables ($K >> m$), whose solution allows one to obtain the current values of $\mu_i^*$.

Consider several examples. Choose as a test model modified oscillator with inertial nonlinearity (Anishchenko-Astakhov oscillator) [16]:

$$\begin{aligned} \frac{dx}{dt} &= mx + y - xz, \\ \frac{dy}{dt} &= -x, \\ \frac{dz}{dt} &= -gz + 0.5g(x + \mid x \mid)x. \end{aligned} \quad (7)$$

Bearing in mind that the signal from oscillator used as a carrier is one-dimensional realization $y(t)$, we realize the following variable substitution

$$Y = y, \ Z = -x, \ X = -mx - y + xz. \quad (8)$$

The equations of oscillator (7) will now have the form

$$\frac{dY}{dt} = Z, \quad \frac{dZ}{dt} = X, \quad \frac{dX}{dt} = f(X, Y, Z), \quad (9)$$

$$\begin{aligned} f(X,Y,Z) \ &= \ \frac{X(X+Y)}{Z} + (mg-1)Z - g(X+Y) \\ &\quad + 0.5g(\mid Z \mid - Z)Z^2. \end{aligned} \quad (10)$$

To transmit several messages in a single carrier, both parameters $m$ and $g$ were modulated simultaneously: $m$ by a chaotic and $g$ by a harmonic signals. To test the used method of demodulation for its robustness we added a normally distributed random value with variance $10^{-5}$ both to the parameters ($g(t)$ and $m(t)$) and to the right-hand sides of Eq. (7). The results are presented in Fig. 1.

Consider the well-known Rössler equations [17]:

$$\begin{aligned} \frac{dx}{dt} &= -y - z, \\ \frac{dy}{dt} &= x + ay, \\ \frac{dz}{dt} &= b + z(x - c). \end{aligned} \quad (11)$$

By analogy with the above example choose $y(t)$ as a carrier. To transform the Eq. (11) into the form (9) one needs to substitute the variables as follows

$$Y = y, \quad Z = x + ay, \quad X = ax + (a^2-1)y - z. \quad (12)$$

Nonlinear function $f$ will have the following form:

$$\begin{aligned} f(X,Y,Z) &= -b + (a-c)X - cY + (ac-1)Z \\ &\quad -aY^2 - aZ^2 - aXY + XZ + (a^2+1)YZ. \end{aligned} \quad (13)$$

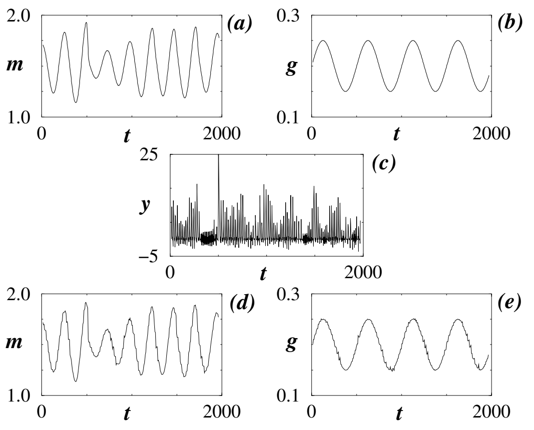


Figure 1: $(a)$ and $(b)$ Information signals, $(c)$ signal in the channel of communication, $(d)$ and $(e)$ restored signals.

First, consider the case when only one parameter is modulated and the other two parameters are fixed ($a = 0.15$, $b = 0.2$). We use as an information signal a certain sequence of symbols corresponding to the letters of English language. The interval of variation of $c(t)$ was split into 32 ranges for this purpose each of them corresponding to some letter or special symbol. The results are present in Fig. 2. To test the used method for its robustness we have also added noise to equations (11).

Let us change all the three parameters of Rössler system. As one can see from Fig. 3, in this example the applied method also demonstrates its effectiveness. The parameter modulation according to the stepwise laws gives the possibility to transmit a color image. In this case the modulating signals carry information about the tints of three basic colors (red, blue and green) forming the image.

The method discussed above is only one possible way for encoding messages. Note, that the potential possibilities of the global reconstruction technique are not restricted by this method. For example, one can suggest several modifications of the given method for multichannel communication.

Return to the Rössler system and add external forces to the right-hand parts of Eq. (11)

$$\begin{aligned} \frac{dx}{dt} &= -y - z + A_1(t), \\ \frac{dy}{dt} &= x + ay + A_2(t), \\ \frac{dz}{dt} &= b + z(x - c) + A_3(t). \end{aligned} \tag{14}$$

Let $A_j(t)$ be now information signals (they must satisfy the requirements of slow time dependence in comparison to the carrier $x_i(t)$). The necessary variable substitutions and nonlinear function $f(X, Y, Z)$ will be of the following form:

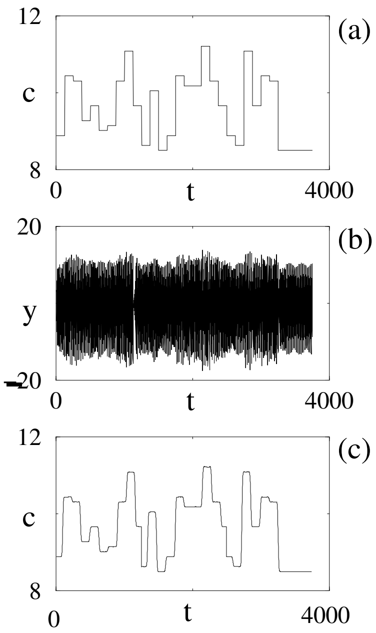


Figure 2: $(a)$ Information signal (a word combination "confidential communication"), $(b)$ carrying chaotic signal, $(c)$ restored signal.

$$\begin{gathered} Y = y, \quad Z = x + ay + A_2, \\ X = ax + (a^2 - 1)y - z + A_1 + aA_2. \end{gathered} \tag{15}$$

$$\begin{aligned} f(X, Y, Z) = {} & C_1 + C_2 X + C_3 Y + C_4 Z + C_5 XY \\ & + C_6 XZ + C_7 YZ + C_8 Y^2 + C_9 Z^2, \end{aligned} \tag{16}$$

where

$$\begin{aligned} C_1 &= A_1 A_2 - A_3 + cA_1 - b \\ C_2 &= a - c - A_2 \\ C_3 &= aA_1 - A_2 - c \\ C_4 &= ac + aA_2 - A_1 - 1 \\ C_5 &= -a \\ C_6 &= 1 \\ C_7 &= 1 + a^2 \\ C_8 &= -a \\ C_9 &= -a. \end{aligned} \tag{17}$$

The values of $C_j$ may be computed numerically at each time moment $i\Delta t$. Then we can obtain the current values of $A_j(t)$ from $C_2$, $C_3$ and $C_1$, i.e., restore all information signals.

Another way for encoding messages may consist in combination of the methods mentioned above: a part

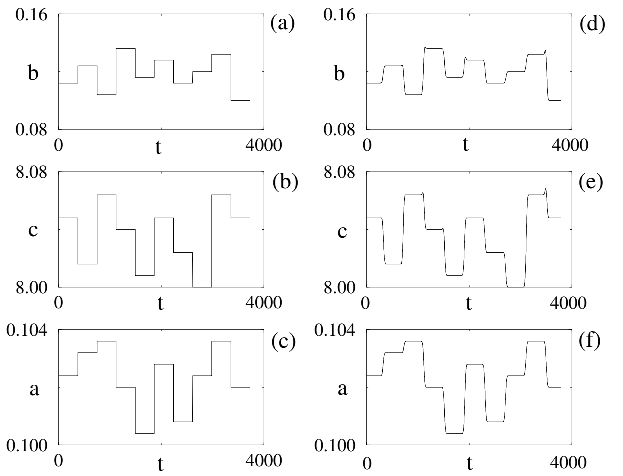


Figure 3: $(a)$, $(b)$, $(c)$ Information signals, $(d)$, $(e)$, $(f)$ restored signals. The signal transmitted via the communication channel is similar to Fig. 2(b).

of information signals perform modulation of the parameters and other part is added as external forces. For example, let $A_2(t) = 0, A_3(t) = 0$. Consider 4 information signals $a(t)$, $b(t)$, $c(t)$ and $A_1(t)$. As one can see from (17) they may be defined at each time moment $i\Delta t$ from $C_j$ (in the following order: $a$ from $C_5 \div C_9$, $c$ from $C_2$, $A_1$ from $C_3$ and $b$ from $C_1$).

Of course, other combinations may also be used.

To realize the proposed method for signal demodulation in practice, the recipient of the information must possess a specialized processor or computer and an ADC. We suppose that such a method for private communication may find its effective application for the exchange of information on short enough distances using cable network, so that noise would not be created while broadcasting.

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