Global reconstruction in the presence of *a priori* information.

Vadim S. Anishchenko, Alexey N. Pavlov, Natalia B. Janson

Nonlinear Dynamics Laboratory, Department of Physics, Saratov State University, Astrakhanskaya St. 83, 410026, Saratov, Russia

ABSTRACT

We discuss a particular case of the global reconstruction problem consisting in the knowledge of the general form of equations of dynamical system which produce the signal under observation. We suggest the application of the reconstruction technique to solve different problems, such as multichannel confidential communication, prediction of the regime of system's functioning on its one-dimension realization, tuning of characteristics of a working oscillator with respect to the ones of standard oscillator. We demonstrate the workability of the parameter determination method in the presence of noise of different origin.

I. INTRODUCTION.

One of the research problems in modern nonlinear dynamics is reconstruction of dynamical systems (DS). A great interest to this problem was caused by the works (Packard et al., 1980; Takens, 1981), where the possibility to restore the phase portrait of a DS on its one-dimension realization $a(t)$ was proved and the condition upon a minimal necessary embedding dimension was formulated.

The works (Packard et al.,1980; Takens, 1981) changed the traditional notions of dynamical systems theory. Before it has been supposed that to study the dynamics of the system in terms of phase space one should know all its phase coordinates. Now a scalar time series is quite enough for this purpose. As a consequence, the possibility has appeared to extract the metrical (dimension) (see, Grassberger and Procaccia, 1983a; 1983b) and dynamical (Lyapunov exponents) (see, Eckmann et al., 1986; Wolf et al., 1985; Sano and Sawada, 1985; Brown, 1993; Parlitz, 1992) characteristics of original attractor from $a(t)$, sampled with some step Δt, i.e., scalar time series $a_i = a(i\Delta t),\ i = 1, \ldots, N$, $N = t_0/\Delta t$, where t_0 is the observation time. At present the methods for computation of these characterictics appear to be standard techniques of data processing similarly to spectral analysis.

The works (Packard et al.,1980; Takens, 1981) were a precondition for solving on their base the problems of prediction of system's state by means of local approximation technique (see, for example, Farmer and Sidorovich, 1987; Casdagli, 1989). In 1987, Cremers and Hübler (Cremers and Hübler, 1987) and independently Crutchfield and McNamara (Crutchfield and McNamara, 1987) proposed an algorithm for global reconstruction, i.e., the method for writing the equations of motion allowing one to obtain a dynamical description for the signal under study in the form of a system of ordinary differential equations (ODE) or a discrete map. The ideas of global reconstruction has been developed in articles (Gouesbet and Maquet, 1992; Gouesbet and Letellier, 1994; Breeden and Hübler, 1990; Kadtke, 1995; Brown et al.,1994; Baake et al., 1992; Abarbanel et al., 1990; Janson et al., 1998). Although their authors approach the problem in slightly different ways, all of them suppose construction of a dynamical description in 2 stages.

At the first one a minimal embedding dimension n is determinated by means of, e.g., evaluation of Hausdorff dimension d or any other generalized dimension. In correspondence with (Takens, 1981), the condition $n \geq 2d + 1$ should be satisfied. But, as the paper (Sauer et al., 1991) notes, such a value of n may often be overestimation and in practice the embedding space with a less dimension ($n > d$) may be enough. At the same stage the method for restoration of phase portrait is chosen (a good review of them is given in (Breeden and Packard, 1994)). The most popular are the delay method and the method of successive derivatives. The mathematical description being reconstructed has the form

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x},\ \vec{\mu}) \qquad \vec{x} \in R^n,\ \vec{\mu} \in R^m \tag{1}$$

or

$$\vec{x}_{n+1} = \vec{F}(\vec{x}_n,\ \vec{\mu}), \tag{2}$$

where $\vec{x}$ is the state vector, $\vec{F}$ is the nonlinear vector-function, $\vec{\mu}$ is the parameter vector.

If the mathematical description is searched in the form (1) the method of successive derivatives allows one to simplify the equations as follows:

$$\frac{dx_1}{dt} = x_2, \; \frac{dx_2}{dt} = x_3, \ldots, \; \frac{dx_n}{dt} = f(\vec{x}, \; \vec{\mu}), \tag{3}$$

i.e., the right-hand parts (3) have only one nonlinear function.

At the second stage the general form of nonlinearity $f(\vec{x}, \; \vec{\mu})$ (or in common case vector-function $\vec{F}(\vec{x}, \; \vec{\mu})$) is stated a priori.

Since it is impossible to indicate even an approximate form of nonlinearities (the latter being a consequence of essential nonuniquelity of the inverse problem) the functions $F_j(\vec{x}, \; \vec{\mu})$, $j = 1, \ldots, n$ are presented as an expansion on some basis. In the simplest case they are defined as an polynomial expansion with a finite number of terms, the expansion coefficients being fitted by means of least squares algorithm. Since the convergence of expansion series is not proved in general, the restriction to a certain number of the first expansion terms has no strict mathematical evidence.

In practice, the correspondence between the signal and its approximate dynamical description is analysed by means of computing the time T during which the reconstructed DS may reproduces the initial signal with the required accuracy. In other words, the time T is measured during which one may predict the state of the system after observation of its signal is stopped.

An arbitrariness in the choice of the dynamical description is a serious shortcoming of the method of global reconstruction and the obtained mathematical models need to be carefully treated.

The problem of how the description is connected with real system remains questionable and may be formulated in this case as follows: does the reconstructed model reproduce the structure of the phase space partition into trajectories of the real system at least in the nearest neighbourhood of the restored dynamical regime?

This problem has been touched upon in (Pavlov et al., 1997), where we proposed to study either stationary data or the time series of transient processes in the initial system. Unfortunatelly, this approach allows one only to test the reconstructed model from the viewpoint of adequacy of mathematical description, but doesn't suggest any recipe to construct such descriptions.

Let us suppose, that the explicit form of nonlinear vector-function $\vec{F}(\vec{x}, \; \vec{\mu})$ is known a priori. In this case the problem of global reconstruction consists in determination of unknown values of parameters μ_i. Such a supposition may be treated as a particular case of the problem under study, but it may be effectively applied in a series of practical tasks. The aim of the present work is to illustrate some of them. In Section 2 we suggest the solution of the problem of confidential communication on the base of reconstruction technique. The possibility to predict the parameter values and the functioning regime of a DS on a scalar time series is discussed in Section 3. The tuning of the characteristics of a working oscillator with respect to the regime of a standard one by means of comparing the signals produced by each of them using the same method is given in Section 4.

II. GLOBAL RECONSTRUCTION IN APPLICATION TO CONFIDENTIAL COMMUNICATION.

In recent years, several methods for secure communication were suggested basing on exploiting broadband chaotic oscillations as a carrying (or masking) signal. The authors of pioneering works on this topic (Cuomo and Oppenheim, 1993; Kocarev et al., 1992; Wu and Chua, 1993; Parlitz et al., 1992; Cuomo et al, 1993; Dedieu et al., 1993) used the phenomenon of synchronization to extract the information signal from chaotic one. There were two concrete ways suggested to solve the problem.

The first one (Cuomo and Oppenheim, 1993; Kocarev et al., 1992; Wu and Chua, 1993) is based on chaotic systems synchronization suggested in (Pecora and Carroll, 1990a; 1990b; Carroll and Pecora, 1991). Information signal $c(t)$ is added to the signal $h(t)$ of oscillator G_1 functioning in chaotic regime. At the output of the transmitter a signal is created $s(t) = h(t) + c(t)$, the power of signal $c(t)$ being much less than the power of $h(t)$. In the receiver consisting of two subsystems a synchronous chaotic responce $h^*(t) = h(t)$ is excited being then subtracted from $s(t)$.

The second possibility (Parlitz et al., 1992; Cuomo et al, 1993; Dedieu et al., 1993) also supposes the existence of two identical chaotic oscillators G_1 and G_2 in the transmitter and the receiver, respectively. One of the parameters of G_1 is modulated according to binary law. In case when the corresponding parameters of both oscillators coincide there arise a regime of synchronous oscillations ("1" is at the output of the receiver); in the opposite case there is no synchronization of oscillations ("0" is at the output). This method gives the possibility to transmit the binary code.

Besides exploiting chaotic synchronization one can protect the transmitted information by means of controlling chaos (Haues et al., 1994; Abarbanel and Linsay, 1993).

Later (Parlitz, 1996; Parlitz and Kocarev, 1996; Parlitz et al., 1996) an important problem of multichannel confidential communication was considered, i.e., the possibility of simultaneous transfer of several messages in a single carrier. The suggested way supposed the parameter modulation of the chaotic oscillator G_1 by information signals, the latter being extracted by means of *autosynchronization* phenomenon.

In (Anishchenko and Pavlov, 1998) we suggested an alternative method for multichannel communication using the technique of global reconstruction. Note, that the first attempts to use the method for reconstruction in communication were made in (Gribkov et al., 1995). But the latter work demonstrated the possibility to extract only a single additive influence on the system. (Moreover, besides an a priori knowledge of the model one needed information about how this influence was added to the equations.)

Our method is more universal and at the same time rather simple. By analogy with (Parlitz, 1996; Parlitz and Kocarev, 1996; Parlitz et al.,1996) information signals perform modulation of parameters of a DS. The difference consists in the method for demodulation. Instead of *autosynchronization* phenomenon we use the technique of reconstruction.

Briefly speaking, the idea of the present method consists in the following. Let we have some dynamical chaos oscillator G_1 as a transmitter, whose mathematical model has the form (1). Let us restrict ourselves to the class of DS's, whose equations may be transformed to the form (3) by means of variable substitution. Many well-known models such as Lorenz, Roessler systems, Chua's circuit, etc., (see, for example, (Gouesbet and Letellier, 1994)) satisfy this requirement.

In the tasks of communication in radiophysics information signals are usually low-frequency in comparison with the carrier. Using this circumstance we consider relatively slow parameter μ_i modulation the latter allowing us to transmit m messages simultaneously:

$$\mu_i^*(t) = \mu_i^0 + \mu_i(t), \qquad i = 1, \ldots, m. \tag{4}$$

In (4) μ_i^0 are the constant values of control parameters of oscillator (1), $\mu_i(t)$ are the information signals. The condition of slow modulation may be written as follows:

$$\mid \overline{\frac{d\mu_i}{dt}} \mid \quad << \quad \mid \overline{\frac{dx_j}{dt}} \mid \tag{5}$$

for any i and j.

Taking into account (4), the equations (1) will have the form

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x},\ \vec{\mu}^*(t)). \tag{6}$$

To realize the messages demodulation from the carrier, which is a one-dimension realization $x_1(t)$ of the system (6), the recepient of information has to know the form of nonlinear vector-function $\vec{F}(\vec{x},\ \vec{\mu})$ (or, what is equivalent, the scalar function $f(\vec{x},\ \vec{\mu})$ of system (3), since the equations (1) permit transformation into the form (3) in agreement with the accepted limitations).

The condition (5) allows one to choose the temporal window t^*, inside which the values of μ_i^* are practically constant. The latter means, that nonautonomous properties of system (6) may be not taken into account during the time interval t^*. Shifting the temporal window along the carrier signal $x_1(t)$, the recepient of information who knows the mathematical model of oscillator G_1 may extract the modulation signals $\mu_i(t)$ in real time. In practice, determination of the current values of $\mu_i(t)$ is realized as follows. The signal $x_1(t)$ of oscillator (6) observed during t^* is differentiated n times to compute the temporal dependencies $x_j(t),\ j = 2, \ldots, n$ and the left-hand parts of system (3).

Since the observable is a scalar series of numbers $x_{1,i} = x_1(i\Delta t),\ i = 1, \ldots, n^*, n^* = t^*/\Delta t$ all the derivatives are computed at discrete time moments $i\Delta t$ using approximate formulas for numerical differentiation. As a result, one should solve the following equation

$$\frac{dx_{n,i}}{dt} = f(\vec{x}_i,\ \vec{\mu}^*) \tag{7}$$

to define the parameters μ_i^*. This is an algebraic equation since the value of $dx_{n,i}/dt$ is computed by means of n-times differentiating the initial signal. By writing (7) for $i = 1, \ldots, n^*$ and solving such a system of n^* algebraic equations with m unknown variables by means of least squares algorithm ($n^* >> m$), the recepient of information obtaines the current values of μ_i^*.

Let us illustrate the above on the examples. When the method was described, the systems with continuous time were discussed. However, the same takes place for discrete maps. An advantage of work with dicrete maps is that one may not take into account the errors of numerical differentiation.

That is why the use of such systems may be interesting from the viewpoint of testing the method for multichannel communication on the simplest examples. By analogy with continuous systems introduce the restriction to the mathematical model (2), which should permit transformation into the following form

$$x_{n+1,1} = x_{n,2},\ x_{n+1,2} = x_{n,3}, \dots,\ x_{n+1,n} = f(\vec{x}_n,\ \vec{\mu}). \tag{8}$$

Consider the equations of well-known Henon map

$$x_{n+1} = 1 - ax_n^2 + y_n, \quad y_{n+1} = bx_n. \tag{9}$$

Bearing in mind, that the sequence of numbers $y_i, i = 1, \dots, n^*$ is available for the researcher, rewrite (9) as follows:

$$y_{n+1} = z_n, \quad z_{n+1} = b - az_n^2/b + by_n. \tag{10}$$

In fact, it is enough to know the values of y_n, y_{n+1} and y_{n+2} to determine the unknown parameters a and b. I.e., an a priori information about the general form of mathematical model allows one to solve the global reconstruction problem for a very short scalar time series. As a consequence, the requirement of a slow modulation (5) becomes not so essential.

An example of multichannel communication on the base of Henon map is presented in Fig.1. Two chaotic signals (the coordinates x(t) and y(t) obtained by integrating of Roessler model) perform modulation of the parameters a and b of the system (9). On each iteration the values of a and b change according to the law shown in Fig.1a,b. In fact, the unit of one-size time corresponds to the one integrating step of the ODE's.

Let us turn to the ODE's and choose Rössler equations (Rössler, 1976) as a test model

$$\frac{dx}{dt} = -y - z, \quad \frac{dy}{dt} = x + ay, \quad \frac{dz}{dt} = b + z(x - c). \tag{11}$$

Choose as a carrier the coordinate y of the system (11). Transform the equations (11) into the form (3) by means of variable substitution (Gouesbet and Letellier, 1994)

$$Y = y, \quad Z = x + ay, \quad X = ax + (a^2 - 1)y - z. \tag{12}$$

As a result we obtain the following form of nonlinear function f:

$$f = -b + (a - c)X - cY + (ac - 1)Z - aY^2 - aZ^2 - aXY + XZ + (a^2 + 1)YZ,$$
$$\vec{\mu} = (a, b, c). \tag{13}$$

First, consider the case when only one parameter is modulated. We assume, that the values of a and b are fixed ($a = 0.15$, $b = 0.2$) and the function f contains only one unknown quantity c. Modulate this parameter by choosing as an information signal a certain sequence of symbols corresponding to the letters of English language. For this purpose split the interval of variation of c into 32 ranges each of them corresponding to some letter or special symbol.

In Fig.2a an information signal $c(t)$ is shown encoding a word combination "confidential communication". In Fig.2b the carrying chaotic signal of oscillator (11) is shown containing the given message. The recepient of the signal who knows the concrete form of the mathematical model of the oscillator extracts the information signal by means of reconstruction (Fig.2c).

A few remarks. First, equations (11) were transformed to the form (3) under supposition of a slow modulation (5), the latter allowing us not to take into account the derivatives of parameters. If the parameters change quikly the equation (13) will have a more complicated form containing the terms with da/dt. In the discussed example the parameter a was constant, therefore such terms were absent. If the parameter a is modulated instead of c according to the law in Fig.2a the condition (5) is not satisfied at the moments of switching, the latter leading to the large enough error in defining the current parameter values at these time moments and to the appearance of δ-like peaks corresponding to each moment of switching. However these peaks may be relatively easily filtered.

Second, the peculiarity of global reconstruction technique is that it may be applied to stationary data as well as to realizations of transient processes. Indeed, since the motion on attractor is described by the same equations as transient processes, nonstationarity of the signal in communication channel is not essential for the solution of the problem of determining the current values of μ_i^*.

Third, any method for information transmission should be analysed from the viewpoint of its stability with respect to noise. Therefore, a special attention will be paid further to the workability of the used method in presence of noise.

Let us change all three parameters of Roessler system. As one can see from the Fig.3, in this example the applied method also demonstrates its effectiveness. The parameter modulation according to the stepwise laws gives the possibility to transmit a color image. In this case the modulating signals carry information about the tints of three basic colors (red, blue and green) forming the image.

Consider the workability of the suggested technique under the noise influence. For simplicity, restrict ourselves to the case of only one parameter modulation. In Fig.4 the dependence is presented of relative meansquare error with which the current values of parameter c is determined versus the variance of normally distributed random value added to the parameter (Fig.4a) and to the first equation of Roessler system (Fig.4b). As one can see from Fig.4, the parameter c is computed with the error of about 1 % while noise exceeds by several orders the real thermal one and the shot noise of an electronic device. The effectiveness of this method in the case of modulation of more than one parameter in the presence of noise was also mentioned in (Anishchenko and Pavlov, 1998).

To realize the described technique for signal demodulation in practice, the recepient of the information must possess a specialized processor or computer and an ADC. We suppose that such method for confidential communication may find its effective application for the exchange of information on short enough distances using cable network, so that it would not create noise while broadcasting.

III. PREDICTING THE VALUES OF PARAMETERS AND THE FUNCTIONING REGIME OF DS.

Let us consider the dynamical system (1) and suppose that the parameters μ_i depend on time but they are varied slowly in comparison with the basic frequency of oscillations of phase variables x_j. Such a situation is not very often encountered in radiohysics but is rather typical for the dynamics of open systems being under the influence of environment. An example is a model of the "predator-prey" type if one supposes that the conditions of existence and nutrition change in time for the "prey" population.

Suppose that the mathematical model (1) is known a priori and the process of parameter changing is deterministic. Like before, the model (1) should permit transformations into the form (3). Under these suppositions one can solve the following problem.

Let the researcher be able to measure the temporal dependence of one phase variable of the system (1) during the observation time t_0. The knowledge of the general form of mathematical model allows one to predict the values of system's parameters for the times $t_0 < t < t_0 + T$, where T is the prediction time. Unlike the traditional problem of the DS's state prediction the possibility for forecasting the parameter values allows one to define the functioning regime of the system after the end of observation of its signal (if the parameter variation leads to the bifurcations of a DS solution).

In practice, the idea mentioned above may be realized in 2 stages. On the first stage, similarly to the problem of confidential communication the knowledge of general form of mathematical model gives the possibility to restore the signals $\mu_i(t)$.

On the second stage, the temporal dependencies $\mu_i(t)$ are observed as the initials realizations for solving the problem of modeling by means of global reconstruction.

The aim is to construct some systems of ODE's allowing one to describe the signals $\mu_i(t)$. Integrating these models with different initial conditions and statistical processing of the results allows one to estimate the time value T during which one may predict the vector $\vec{\mu}$ with the required accuracy.

Let us illustrate everything mentioned above using as an example a model of modified oscillator with inertial nonlinearity (Anishchenko-Astakhov oscillator) (Anishchenko and Astakhov, 1983; Anishchenko, 1995):

$$\frac{dx}{dt} = mx + y - xz, \quad \frac{dy}{dt} = -x, \quad \frac{dz}{dt} = -gz + 0.5g(x + \mid x \mid)x. \tag{14}$$

Choose the dependence $y(t)$ as an observable. As it is shown in (Anishchenko and Pavlov, 1998), the equations of oscillator (14) may be written in the form

$$\frac{dY}{dt} = Z, \quad \frac{dZ}{dt} = X, \quad \frac{dX}{dt} = f(Y, Z, X, \vec{\mu}), \quad \vec{\mu} = (m, g), \tag{15}$$

$$f = \frac{X(X+Y)}{Z} + (mg-1)Z - g(X+Y) + 0.5g(\mid Z \mid - Z)Z^2 \tag{16}$$

by means of variable substitutions:

$$Y = y, \quad Z = -x, \quad X = -mx - y + xz. \tag{17}$$

Let us modulate the parameter m in (14) by chaotic signal obtained by integrating of Roessler system where the time renormalization was performed to prove the condition of slow modulation. A researcher who knows the general form of the model (14) can restore the signal $m(t)$ by measuring the time dependence of only one phase variable y (Fig.5, solid curve) by analogy with the problem of confidential communication.

Bearing in mind that the equations of the system which produce the signal $m(t)$ are unknown apply the method for global reconstruction to obtain an approximate dynamical description of the given signal. The solution of reconstructed mathematical model is also presented in Fig. 5 (dashed curve).

In the given example the prediction time for the parameter m value with precision 5% equals to approximatelly 3 basic periods of oscillations of Roessler system with renormalized time.

IV. TUNING OF A WORKING OSCILLATOR WITH RESPECT TO THE FUNCTIONING REGIME OF STANDARD ONE USING ONE-DIMENSION REALIZATION.

Using the same technique of a DS parameter value determination on its realization having an a priori information about the general form of mathematical model the analog of "phase-locked loop" may be suggested.

Consider a block-sheme presented in Fig.6 where the following designations are used: G_1 is an standard oscillator whose parameters are changed in time; G_2 is a working oscillator; 1 is the sheme of comparison being a specialized processor or computer; 2 is a block of control which realize the tuning of parameters of oscillator G_2.

The way of functioning of such a sheme is very simple. The signals from standard and working oscillator come to the sheme of comparison where the parameter vectors of both of them are determined. The difference signals come to the block 2 which realizes a tuning of oscillator G_2 with respect to the regime of G_1.

Besides the parameter tuning the tuning of phase variables may be carried out from time to time to avoid functioning of G_2 with the same parameters as G_1 but in another dynamical regime (in the case of multistability).

V. CONCLUSION.

In the present work a particular case of the global reconstruction problem was discussed consisting in the presence of a priori information about the dynamical system producing the signal under study. The technique of DS's parameter determination on one-dimension realization is used to solve several applied problems such as multichannel confidential communication, prediction of the functioning regime of the system, tuning of the characteristics of a working oscillator with respect to the ones of a standard oscillator. In spite that the practical importance of the two latter problems is not high enough, they were included into the present paper to illustrate how the method for global reconstruction in the presence of a priori information may find its application while solving very different problems.

Note, that the global reconstruction technique supposes the work with scalar time series. Therefore, the precision of parameter computation depends on the errors of numerical differentiation, which are defined by the charactericties of analog-to-digital converter (ADC). It is clear that to compute μ_i with high precision one needs the ADC with high resolution. In our numerical simulations we used 4-byte numbers and the sampling step $\Delta t = 0.025$ sec.

In the present work we suggested a series of applications of the reconstruction algorithm, and on our opinion the problem of multichannel communication is the most interesting of them. In Section 2 another method for multichannel communication based on the *autosynchronization* phenomenon was mentioned (Parlitz, 1996; Parlitz and Kocarev, 1996; Parlitz et al.,1996).

Not being aimed to compare the suggested technique with the one suggested in (Parlitz, 1996; Parlitz and Kocarev, 1996; Parlitz et al.,1996) (this may be a topic of a separate research), note, that the approach of a global reconstruction does not impose any limitations to the equations (1) except the possibility to be reduced to the form (3). I.e., the nonlinear function $f(\vec{x},\ \vec{\mu})$ may be either smooth or containt discontinuities of derivatives. Moreover, the requirement of a slow parameter modulation is not always essential for workability of the reconstruction technique. If this requirement is fulfilled for parameter a of Roessler system, communication by means of modulation of other two parameters (b and c) of the same model (if $y(t)$ is chosen as a carrier) allows one to increase the frequency range of information signals.

In the simplest case when only one parameter is being modulated while the other ones are fixed (for example, we modulate only parameter b of the system (11)) to define the current value of $b(t)$ at discrete time moments $i\Delta t$ one needs to know the phase coordinates and derivatives of them at only one (the same) time moment. The necessity to work with temporal window is connected with the influence of errors of numerical differentiation. It is impossible to avoid them completely, but the use of approximation of the parameter value inside the temporal window by the least squares algorithm allows one to decrease their influence. The size of the window t^* is defined from the ratio of basic frequences of signals $b(t)$ and $y(t)$ (the parameter $b(t)$ should remain practically constant during t^*) and in practice may be less as one basic period of $y(t)$.

In the present work the technique for multichannel communication was analysed using the Roessler model as an example. It was also tested on the series of other model systems in the presence of noise of different origin and demonstrated its workability and high effectiveness.

The work was partly supported by INTAS grant (INTAS 96-0305).

References:

Abarbanel, H.D.I., Brown, R. and Kadtke, J.B. (1990). Prediction in chaotic nonlinear systems: methods for time series with broadband Fourier spectra. Phys. Rev. A. **41**, 1782-1807.

Abarbanel, H.D.I. and Linsay, P.S. (1993). Secure communications and unstable periodic orbits of strange attractors. IEEE Trans. Circuits Syst. **40**, 643-645.

Anishchenko, V.S. (1995) Dynamical Chaos – Models and Experiments, edited by L. Chua, World Scientific, Singapore. 400 pp.

Anishchenko, V.S. and Astakhov V.V. (1983). Eksperimentalnoe issledovanie mehanizma vozniknovenija i struktury strannogo attraktora v generatore s inertsionnoi nelineinost'ju. Radiotehnika i elektronika (in Russian). **28**, No.6, 1109-1115.

Anishchenko, V.S. and Pavlov, A.N. (1998). Global reconstruction in application to multichannel communication. Phys. Rev. E. **57** (accepted for publication in No.1.)

Baake, E., Baake, M., Bock, H.G. and Briggs, K.M. (1992). Fitting ordinary differential equations to chaotic data. Phys. Rev. A. **45**, 5524-5529.

Breeden, J.L. and Hübler, A. (1990). Reconstructing equations of motion from experimental data with unobserved variables. Phys. Rev. A. **42**, 5817-5826.

Breeden, J.L. and Packard, N.H. (1994). A learning algorithm for optimal representation of experimental data. Int. J. Bif. Chaos. **4**, 311-326.

Brown, R. (1993). Calculating Lyapunov exponents for short and/or noisy data sets. Phys. Rev. E. **47**, 3962-3969.

Brown, R., Rulkov, N.F. and Tracy, E.R. (1994). Modeling and synchronizing chaotic systems from time-series data. Phys. Rev. E. **49**, 3784-3800.

Carroll, T.L. and Pecora, L.M. (1991). Synchronizing chaotic circuits. IEEE Trans. Circuits Syst. **38**, 453-456.

Casdagli, M. (1989). Nonlinear prediction of chaotic time series. Physica D. **35**, 335-356.

Cremers, J. and Hübler, A. (1987). Construction of differential equations from experimental data. Z. Naturforschung A. **42**, 797-802.

Crutchfield, J.P. and McNamara, B.S. (1987). Equations of motion from a data series. Complex Systems. **1**, 417-452.

Cuomo, K.M. and Oppenheim, A.V. (1993). Circuit implementation of synchronized chaos with applications to communications. Phys. Rev. Lett. **71**, 65-68.

Cuomo, K.M., Oppenheim, A.V. and Strogatz, S.H. (1993). Synchronization of Lorenz-based chaotic circuits with application to communications. IEEE Trans. Circuits Syst. **40**, 626-633.

Dedieu, H., Kennedy, M.P. and Hasler, M. (1993). Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuit. IEEE Trans. Circuits Syst. **40**, 634-642.

Eckmann, J.-P., Kamphorst, S.O., Ruelle, D. and Ciliberto, S. (1986). Liapunov exponents from time series. Phys. Rev. A. **34**, 4971-4979.

Farmer, J.D. and Sidorowich, J.J. (1987). Predicting chaotic time series. Phys. Rev. Lett. **59**, 845-848.

Gouesbet, G. and Maquet, J. (1992). Construction of phenomenological models from numerical scalar time series. Physica D. **58**, 202-215.

Gouesbet, G.and Letellier, C. (1994). Global vector-field reconstruction by using a multivariate polynomial L_2 approximation on nets. Phys. Rev. E. **49**, 4955-4972.

Grassberger, P. and Procaccia, I. (1983a). Characterization of strange attractors. Phys. Rev. Lett. **50**, 346-349.

Grassberger, P. and Procaccia, I. (1983b). Measuring the strangeness of strange attractors. Physica D. **9**, 189-208.

Gribkov, D.A., Gribkova, V.V., Kuznetsov, Y.I. and Rzhanov, A.G. (1995). Global dynamical modeling of time series and applications to restoration of broadband signal characteristics, in *Chaotic, Fractal and Nonlinear Signal Processing, Mystic, Juli, 1995*, ed. R.A. Katz, (AIP Conf. Proc. **375**, AIP Press, New York), pp. 181-188.

Hayes, S., Grebogi, C., Ott, E. and Mark, A. (1994). Experimental control of chaos for communication. Phys. Rev. Lett. **73**, 1781-1784.

Janson N.B., Pavlov A.N. and Anishchenko V.S. (1998). One method for restoring inhomogeneous attractors. Int. J. Bif. Chaos. **8** (accepted for publication in No.4)

Kadtke, J. (1995). Classification of highly noisy signals using global dynamical models. Phys. Lett. A. **203**, 196-202.

Kocarev, L., Halle, K.S., Eckert, K., Chua, L.O. and Parlitz, U. (1992). Experimental demonstration of secure communications via chaotic synchronization. Int. J. Bif. Chaos. **2**, 709-713.

Packard, N.H., Crutchfield, J.P., Farmer, J.D. and Shaw, R.S. (1980). Geometry from a time series. Phys. Rev. Lett. **45**, 712-715.

Parlitz, U. (1992). Identification of true and spurious Lyapunov exponents from time series. Int. J. Bif. Chaos. **2**, 155-165.

Parlitz, U. (1996). Estimating model parameters from time series by autosynchronization. Phys. Rev. Lett. **76**, 1232-1235.

Parlitz, U., Chua, L.O., Kocarev, L., Halle, K.S. and Shang, A. (1992). Transmission of digital signals by chaotic synchronization. Int. J. Bif. Chaos. **2**, 973-977.

Parlitz, U. and Kocarev, L. (1996). Multichannel communication using autosynchronization. Int. J. Bif. Chaos. **6**, 581-587.

Parlitz, U., Kocarev, L., Stojanovski, T. and Preckel, H. (1996). Encoding messages using chaotic synchronization. Phys. Rev. E. **53**, 4351-4361.

Pecora, L.M. and Carroll, T.L. (1990). Synchronization in chaotic systems. Phys. Rev. Lett. **64**, 821-824.

Pecora, L.M. and Carroll, T.L. (1991). Driving systems with chaotic signals. Phys. Rev. A. **44**, 2374-2383.

Pavlov A.N., Janson N.B. and Anishchenko V.S. (1997). Statistical approach to the problem of global reconstruction, in *Procedings of Int. Conference COC'97, St.Peterburg, August 1997*, eds. F.L. Chernousko and A.L. Fradkov, **2**, 315-316.

Rössler, O.E. (1976). An equation for continuous chaos. Phys. Lett. A. **57**, 397-398.

Sano, M. and Sawada, Y. (1985). Measurement of the Lyapunov spectrum from a chaotic time series. Phys. Rev. Lett. **55**, 1082-1085.

Sauer, T., Yorke, J.A. and Casdagli, M. (1991). Embedology. Journal of Statistical Physics. **65**, 579-616.

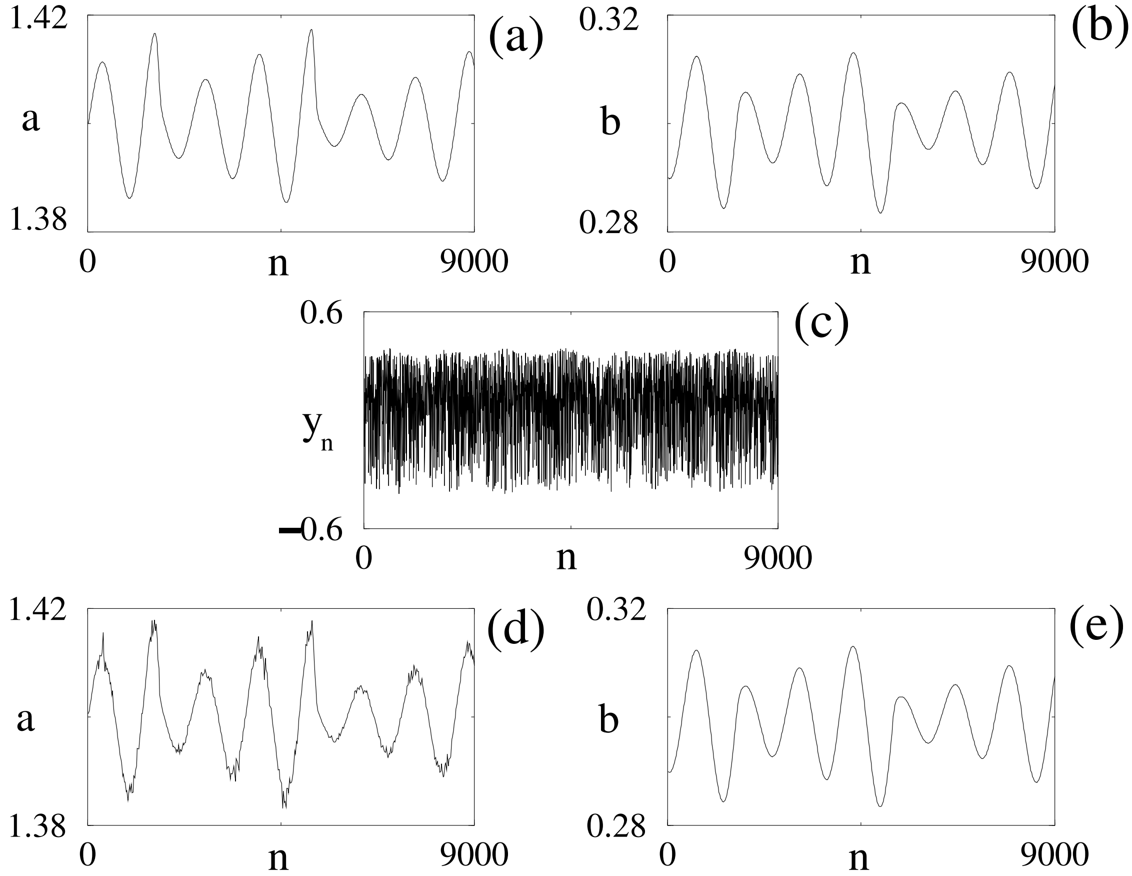
Takens, F. (1981) Detecting strange attractors in turbulence, in *Dynamical Systems and Turbulence, Warwick 1980*, eds. D. Rang and L.S. Young, Lecture Notes in Mathematics **898**, (Springer, Berlin), pp. 366-381.

Wolf, A., Swift, J.B., Swinney, H.L. and Vastano, J.A. (1985). Determining Lyapunov exponents from a time series. Physica D. **16**, 285-317.

Wu, C.W. and Chua, L.O. (1993). A simple way to synchronize chaotic systems with applications to secure communication systems. Int. J. Bif. Chaos. **3**, 1619-1627.

Figure captions:

Fig.1.
a,b - chaotic signals, performing the modulation of the parameters a and b of Eqs.(9);
c - the sequence y_n of Eqs.(9);
d,e - the restored signals.
Fig.2
a - an information signal;
b - the carriyng chaotic signal;
c - the restored signal.
Fig.3
a,b,c - the information signals;
d,e,f - the restored signals.
Fig.4
The dependence of relative meansquare error of determination of the value of parameter c (in %) versus the variance of normally distributed random value added additively to the parameter (a) and to the first equation of Roessler system (b).
Fig.5
The restored signal $m(t)$ (solid curve) and the solution of reconstructed mathematical model (dashed curve).
Fig.6
Block-sheme of the analog of "phase-locked loop".



(a)
1.42
a
1.38
0
n
9000
(b)
0.32
b
0.28
0
n
9000
(c)
0.6
y_n
−0.6
0
n
9000
(d)
1.42
a
1.38
0
n
9000
(e)
0.32
b
0.28
0
n
9000

Fig. 1

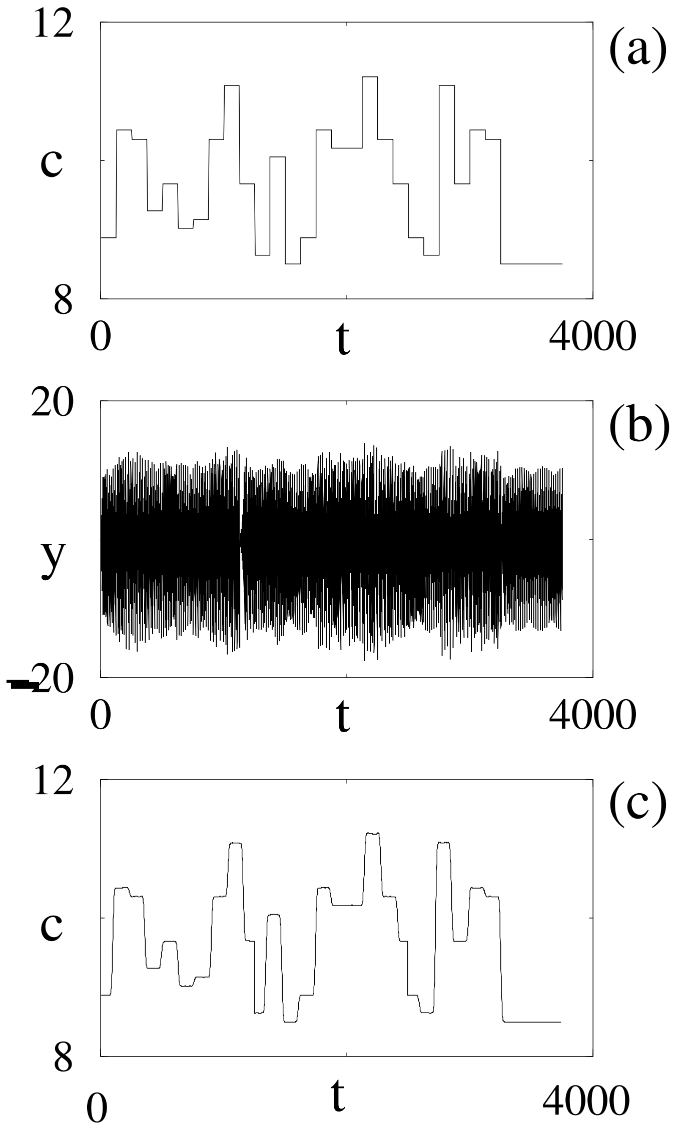


Fig. 2

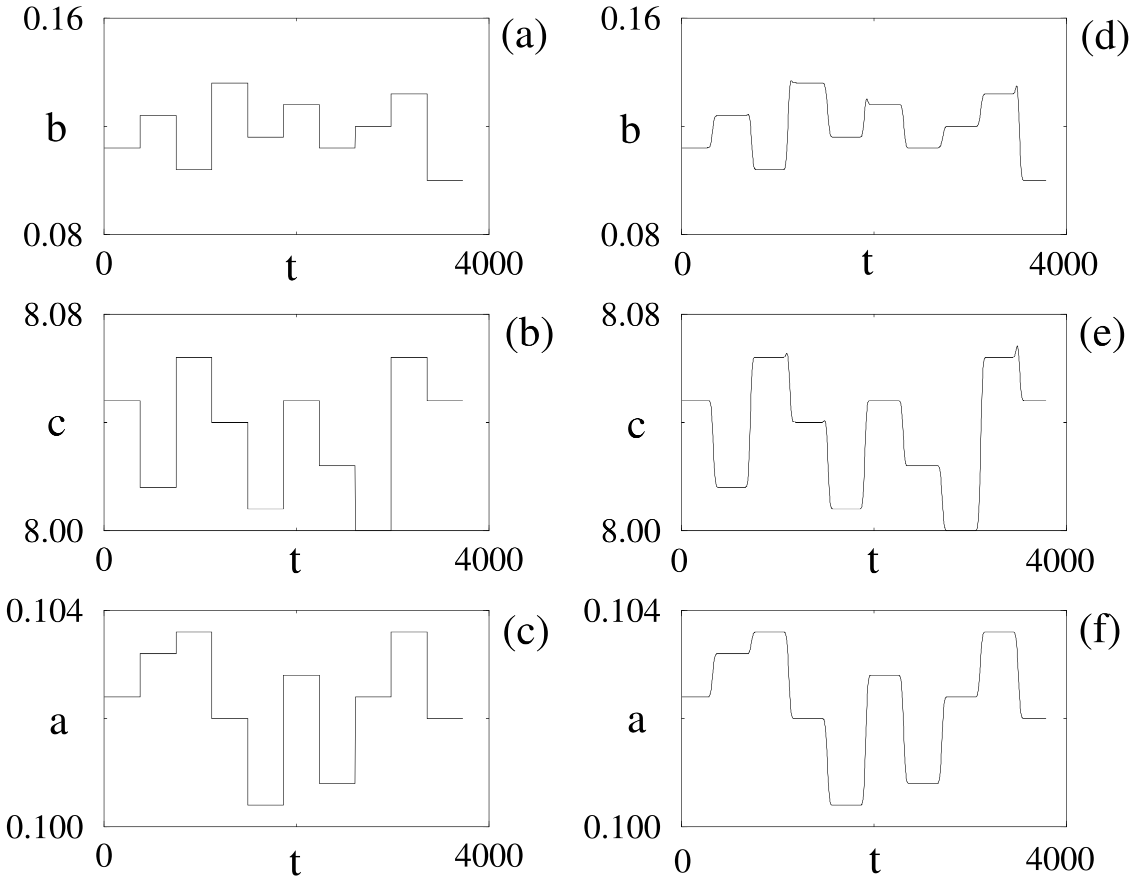

0.16
(a)
b
0.08
0
t
4000
0.16
(d)
b
0.08
0
t
4000
8.08
(b)
c
8.00
0
t
4000
8.08
(e)
c
8.00
0
t
4000
0.104
(c)
a
0.100
0
t
4000
0.104
(f)
a
0.100
0
t
4000

Fig. 3

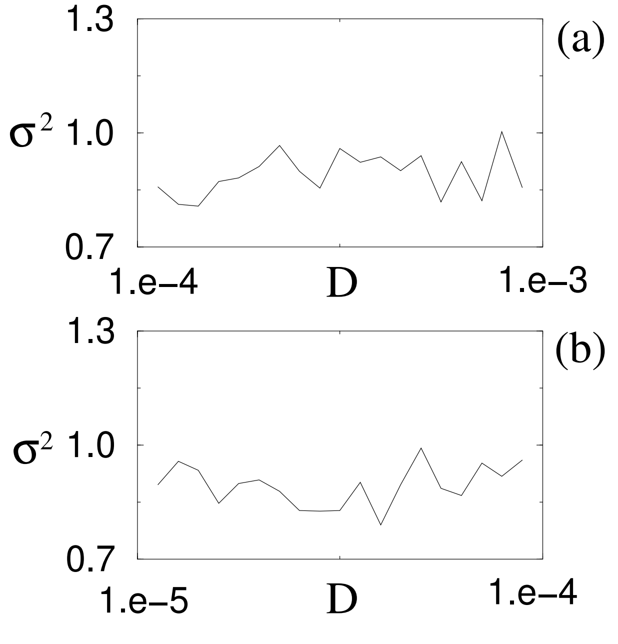

1.3
(a)
σ² 1.0
0.7
1.e−4
D
1.e−3
1.3
(b)
σ² 1.0
0.7
1.e−5
D
1.e−4

Fig. 4

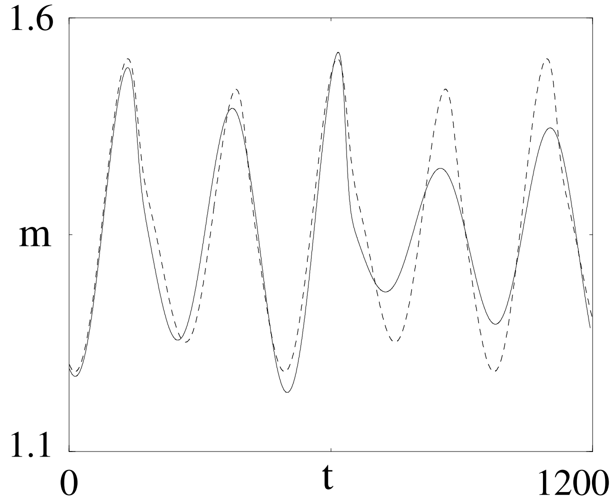


Fig. 5

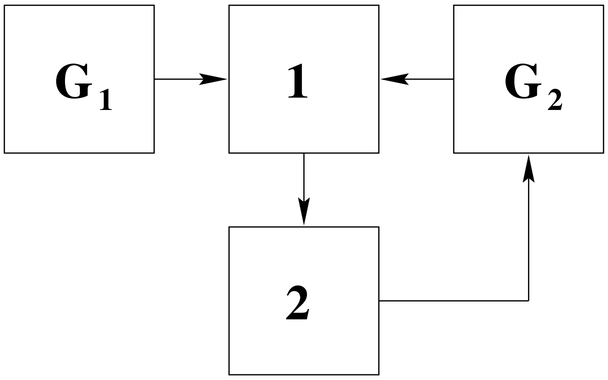


Fig. 6