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Wavelet Analysis of the Structure of Point Processes

A. N. Pavlov* and O. N. Pavlova
Saratov State University, Saratov, 419026 Russia
** e-mail: pavlov@chaos.ssu.runnet.ru*

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Abstract—The structure of point processes generated by a threshold system can reflect interactions of the intrinsic dynamics of this system and that caused by an external action, in particular, in the form of frequency modulation. A new method based on the double wavelet transform is proposed for the detection of such changes in the structure of nonstationary point processes.

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Investigation into the processing of information in living systems is among the most urgent problems of modern natural sciences. Such studies traditionally involve an analysis of the structure of point processes [1], where the information is contained in the timing of certain events. A classical example is offered by the dynamics of a sensor neuron, which generates characteristic pulses (spikes) when the input signal exceeds a certain threshold. The mechanisms of spike generation are now well known and clearly understood [2]. However, there are still many questions concerning the ways in which neurons and their ensembles transmit information about the environment.

A sensor neuron can be considered as a threshold device that converts an input signal into output spike trains (Fig. 1a). The output spikes have identical shapes and equal amplitudes, so that information concerning the external action can be contained only in the values of interspike intervals (ISIs). The possibility of recovering the characteristics of input signals from the output sequence of spikes (offering an example of the point

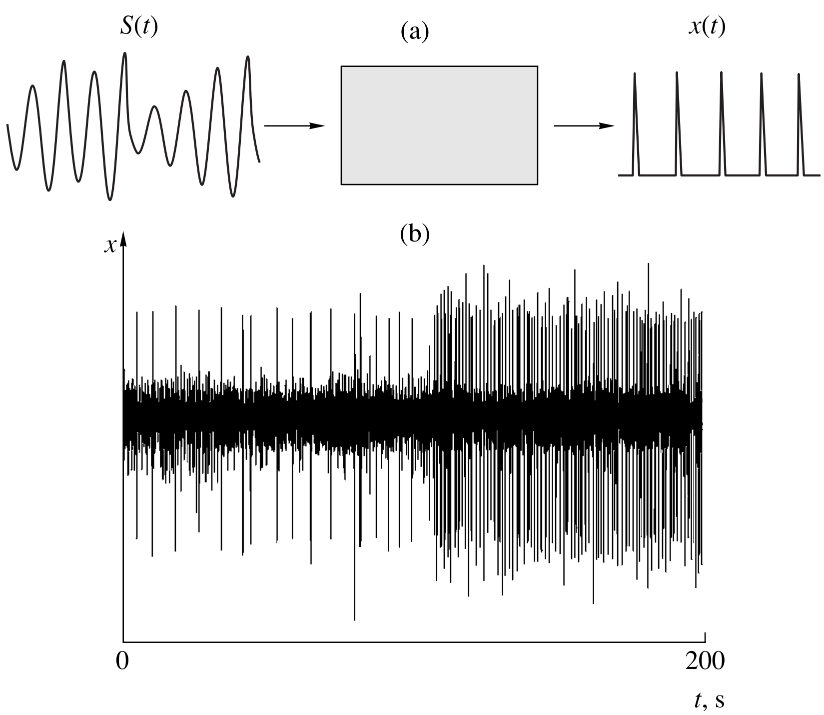

Fig. 1. Conversion of an input signal $S(t)$ into a train of spikes generated by a threshold device at the moments of threshold crossing: (a) schematic diagram; (b) example of an experimental time series of a neuron response.

process) has been extensively studied in recent years [3–8]. Many of these investigations were devoted to the conversion of signals by threshold devices possessing no intrinsic dynamics. In particular, analysis of the classical models of threshold systems such as the integrate-and-fire [3] and threshold-crossing [4, 7] models showed that various characteristics of complex input dynamics are retained in the structure of the output point process [3–7, 9–11].

The situation is considerably complicated if a threshold device exhibits intrinsic dynamics and is capable of generating spikes in the absence of any external action. When an input signal arrives, the external and internal dynamics exhibit superimposition, and the possibility of analyzing the process of signal conversion in this threshold device is not as evident. For example, let us consider the process of information encoding by a real neuron. Figure 1b shows an example of neuronal activity recording in experiments on rats with special microelectrodes implanted into paws. The first part of the record displays the output signal dynamics (featuring rare spikes) in the absence of an external action. The second part shows the dynamics measured under the conditions of an external action, wherein the animal's paw was irritated (by pricking) at a frequency of 1 Hz. As can be seen, this external action produces a change in the output signal structure, which is manifested by an increase in the output spike generation frequency.

In order to establish whether certain characteristic rhythms are inherent in the measured point process $x(t)$ and to assess the stability of such modes, we performed a wavelet transform [12, 13] using the following formula:

$$W(a, b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt. \quad (1)$$

Here, $W(a, b)$ are the wavelet transform coefficients, a is the observation scale, b is the time shift parameter, and ψ is the basis set (wavelet) function; the asterisk denotes complex conjugate. The choice of function ψ is determined by the goals of a particular investigation. Each basis set has definite features in the time and frequency domains and, hence, can better reveal certain properties of the process under consideration.

In investigations of the spectrum of a dynamic process, it is a common practice to use the Morlet basis set functions, which provide good frequency evaluation. This wavelet function can be written in a simple form as

$$\psi(\tau) = \pi^{-1/4}\exp(-j2\pi f_0\tau)\exp\left[-\frac{\tau^2}{2}\right], \quad (2)$$

where f_0 characterizes the frequency resolution, compromising between the wavelet localization in the time and frequency domains. For the range of frequencies considered in this study, a proper choice is $f_0 = 1$.

The analysis of ISIs can be performed using an approach based on the concept of approximated average instantaneous frequency, which was developed in [9, 10]. According to this approach, the average instantaneous frequency is chosen as the point process $x(t)$ in the wavelet transform (1). An alterative is offered by the passage from a time series presented in Fig. 1b to a sequence of delta functions corresponding to the moments of spike generation. In this case, it is possible to calculate the wavelet transform and the signal spectrum by analytical methods.

Let us consider changes in the point process depicted in Fig. 1b, which take place upon the onset of external action. The initial ISI train exhibited rhythms with periods of about 20 and 8 s (Fig. 2a). After the external action, the spectrum exhibits a clear peak at 1 Hz, which is the external signal frequency (Fig. 2b); however, some slow processes are also retained. Changes in the signal structure can be traced in more detail using the frequency–time diagrams obtained by means of wavelet analysis (Figs. 2c and 2d). Initially, the system exhibits several rhythms at frequencies "floating" with the time. These modes are not stable: appearing in some regions of the initial time series, they can vanish when the system exhibits frequency switching. Application of the external action leads to the appearance of a new mode ($\simeq$1 Hz); however, the new rhythm is not constant and exhibits oscillations. This behavior indicates that the system features interaction between the intrinsic neuronal dynamics and that caused by the external action, rather than exhibits a simple reaction of the drive–response type.

The above results allow us to formulate a hypothesis that the process of data encoding by a neutron in some cases can be considered in terms of frequency modulation. Indeed, according to one possible interpretation of Fig. 2d, the induced 1-Hz rhythm is a carrier frequency that is modulated by slow intrinsic processes of the neuronal dynamics. As is known in radio physics, frequency modulation is among the methods of data transfer—and it is quite probable that this method is also employed as a variant of information encoding by neurons.

Proceeding from this hypothesis, it is possible to determine the modulation characteristics (frequency, amplitude) using the double wavelet analysis technique proposed in [14, 15]. According to this method, the time series of the instantaneous frequency variation about 1 Hz is considered as the input signal for another wavelet transform. In this way, we can identify all rhythms participating in the modulation (Fig. 2e) and determine the depth of modulation for each particular process. As can be seen from Fig. 2e, the slow modulating signal qualitatively corresponds to the neuronal

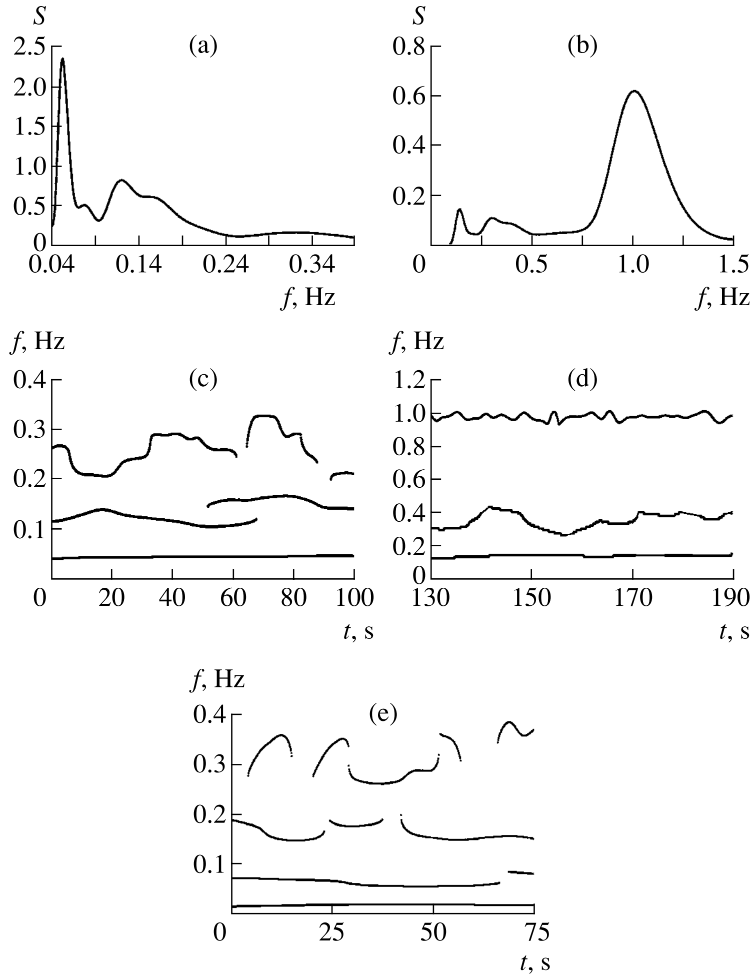

Fig. 2. Typical dynamics of a point process: (a, b) power spectra in the absence and presence of an external periodic 1-Hz action, respectively; (c, d) temporal dynamics of the characteristic modes in the power spectra in the absence and presence of an external periodic action, respectively, as revealed by the wavelet transform; (e) dynamics of slow rhythms producing instantaneous modulation of the 1-Hz rhythm, as revealed by the doublet wavelet analysis.

dynamics observed before the onset of external action (Fig. 2c). Thus, even during the external stimulation, the ISI structure retains the dynamics inherent in the spontaneous neuronal activity.

One possible explanation of the observed behavior is related to the influence of subthreshold dynamics. In the absence of an external action, the threshold-crossing events take place randomly (e.g., due to various fluctuations). When the external signal arrives, the time of threshold crossing can depend on the phase of subthreshold oscillations. Depending on this phase, the threshold crossing can take place slightly earlier or later, which leads to variations in the ISI at the frequency of subthreshold oscillations. In this case, the double wavelet analysis technique offers an effective method for the analysis of point processes, which makes possible the detection of changes in the structure of subthreshold oscillations under the conditions of nonstationary and complicated multimode dynamics.

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REFERENCES

1. T. Sauer, Phys. Rev. Lett. **72**, 3811 (1994).
2. H. C. Tuckwell, *Introduction to Theoretical Neurobiology* (Cambridge Univ. Press, Cambridge, 1988).
3. D. M. Racicot and A. Longtin, Physica D **104**, 184 (1997).
4. R. Castro and T. Sauer, Phys. Rev. E **55**, 287 (1997).
5. T. Sauer, *Nonlinear Dynamics and Time Series*, Ed. by C. Culter and D. Kaplan, Fields Institute Communica-

tions (American Mathematical Society, Providence, 1997), Vol. 11, pp. 63–75.

6. R. Hegger and H. Kantz, Europhys. Lett. **38**, 267 (1997).
7. R. Castro and T. Sauer, Phys. Rev. Lett. **79**, 1030 (1997).
8. M. Ding and W. Yang, Phys. Rev. E **55**, 2397 (1997).
9. N. B. Janson, A. N. Pavlov, A. B. Neiman, and V. S. Anishchenko, Phys. Rev. E **58**, R4 (1998).
10. A. N. Pavlov, O. V. Sosnovtseva, E. Mosekilde, and V. S. Anishchenko, Phys. Rev. E **61**, 5033 (2000).
11. A. N. Pavlov, O. V. Sosnovtseva, E. Mosekilde, and V. S. Anishchenko, Phys. Rev. E **63**, 036205 (2001).
12. A. Grossmann and J. Morlet, SIAM (Soc. Ind. Appl. Math.), J. Math. Anal. **15**, 723 (1984).
13. C. K. Chui, *Wavelets: A Mathematical Tool for Signal Analysis (SIAM Monographs on Mathematical Modeling and Computation* (SIAM, Philadelphia, 1997).
14. O. V. Sosnovtseva, A. N. Pavlov, E. Mosekilde, et al., Phys. Rev. E **70**, 031915 (2004).
15. O. V. Sosnovtseva, A. N. Pavlov, N. A. Brazhe, et al., Phys. Rev. Lett. **94**, 218103 (2005).

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