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Double Wavelet Analysis of Modulation Phenomena in Nonstationary Dynamics

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Abstract—A new method based on the double wavelet transform is proposed for the analysis of features related to the interaction of modes in nonstationary dynamics of systems with several time scales. Examples are presented, which illustrate new possibilities offered by the proposed method in determining the characteristics of amplitude and frequency modulation from the time series of complex oscillatory processes.

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Many natural processes are nonstationary and exhibit strong temporal variations of their characteristics. Classical methods used for analysis of the structure of signals have been mostly developed for the investigation of stationary random processes, and the application of such methods to the analysis of nonstationary data frequently leads to problems in the interpretation of results. For example, the appearance of two peaks at nonmultiple frequencies in the power spectrum of a system can be related to principally different situations, where the system either simultaneously features two independent modes or exhibits frequency switching and only one of the two modes can be detected at each moment of time. Special methods have been developed for the processing of nonstationary data, the best known and most frequently used being wavelet analysis [1–3]. In contrast to classical spectral analysis, this method not only reveals the presence of different characteristic rhythms, but it also allows both instantaneous frequencies and amplitudes of the corresponding rhythmic components to be evaluated and their temporal evolution to be traced.

The wavelet transform of a given signal $x(t)$ is calculated as [4, 5]

$$W(a, b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt, \qquad (1)$$

where $W(a, b)$ are the wavelet transform coefficients, a is the observation scale, b is the displacement parameter, and ψ is the basis set (wavelet) function. In investigations of the rhythmic components (modes) of dynamic processes, it is a common practice to use the Morlet basis set functions,

$$\psi(\tau) = \pi^{-1/4}\exp(-j2\pi f_0\tau)\exp\left[-\frac{\tau^2}{2}\right], \qquad (2)$$

where f_0 characterizes the frequency resolution; the relationship between the scaling parameter a and the frequency f of an analyzed rhythmic component is given by the ratio $f = f_0/a$. The temporal variation of instantaneous frequencies and amplitudes of the characteristic rhythms is studied by determining the local maxima of coefficient $W(a, b)$ at each fixed value of the displacement parameter $b = b^*$.

If the dynamics of a system simultaneously exhibits several independent modes, these components can be involved in various interactions, in particular, with synchronization of oscillations [6]. Another example is offered by the modulation of the amplitude or frequency of a fast mode by slower modes. This interaction can be readily revealed by an analysis of the fast (modulated) dynamics. In practice, however, this situation may encounter difficulties if only the slowly varying component (in which the fast dynamics is hidden) can be measured in experiment. The problem is even more complicated under conditions of nonstationary rhythms, where the frequency and/or amplitude of modulation are subject to temporal variations. If these characteristics exhibit strong variations, difficulties can arise in the separation of rhythmic components from the time series by means of the band filtration procedure. Indeed, a transmission band for the fast mode separation can be selected as neither too narrow (because of the nonstationary character) nor too wide (in order to

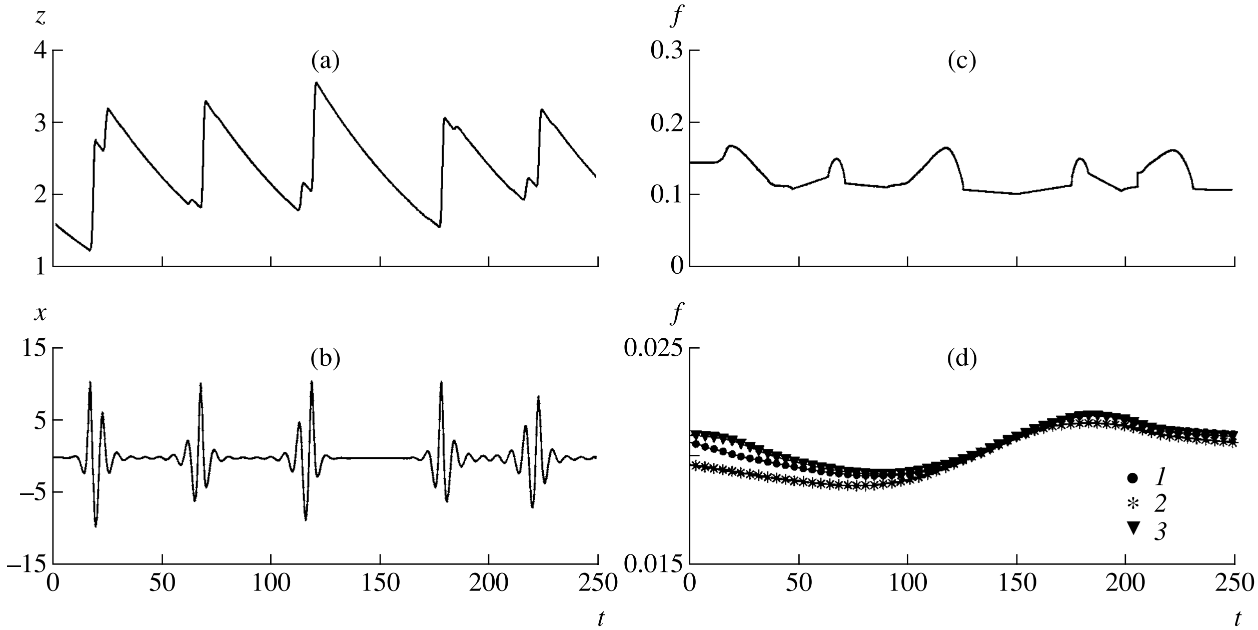

Fig. 1. Typical dynamics of a model generator of chaotic radio signals described by Eqs. (3): (a, b) time series of $x(t)$ and $z(t)$ variables in the automodulation regime ($k = 2.90328$, $g = 0.012505$, $b = 5 \times 10^{-5}$); (c) instantaneous frequency of a fast mode revealed by wavelet transform of the $z(t)$ signal; (d) temporal variation of the instantaneous frequencies of (*1*) the slow mode and the (*2*) amplitude and (*3*) frequency modulation of the fast mode (curve *1* was obtained using wavelet transform of $z(t)$ variable, and curves *2* and *3* were obtained by double wavelet transform).

eliminate harmonics of the slow mode). In such cases, the mode dynamics is expediently studied by means of wavelet analysis based on relation (1).

In this Letter, we describe a special approach employing the double wavelet technique [7, 8], which is intended to reveal features in the amplitude and frequency modulation of oscillations under nonstationary conditions. The idea of the proposed approach consists in using the separated instantaneous frequencies or amplitudes as the initial signal for another, secondary wavelet transform (1), which provides information about all components involved in the modulation of fast dynamics by slow processes.

Let us consider the possibilities provided by the proposed approach on several examples. The first example is the model of a generator of chaotic radio signals (oscillator with inertial nonlinearity) [9]:

$$\begin{aligned} \dot{x} &= kx + y - xz - bx^3, \\ \dot{y} &= -x, \\ \dot{z} &= -gz + gx(x + |x|)/2. \end{aligned} \qquad (3)$$

By varying the control parameters of system (3), it is possible to obtain various regimes, including that with automodulation, which is characterized by relatively slow oscillations of $z(t)$ and faster dynamics of $x(t)$ and $y(t)$. In order to complicate the task, let us consider the case of a nonstationary dynamics (transient chaos) with the time series depicted in Figs. 1a and 1b. Investigation of the $z(t)$ signal structure by means of the double wavelet transform allows the instantaneous frequency and amplitude of the fast mode to be determined after the primary transformation of the slow variable (Fig. 1c). Then, the secondary wavelet transform yields the instantaneous frequencies of the amplitude and frequency modulation. As can be seen from Fig. 1d, these frequencies virtually coincide with the instantaneous frequency of slow dynamics in $z(t)$. Thus, it can be ascertained that the double wavelet analysis of slow phase variables under the conditions of interacting modes provides information about the characteristics of modulation and their temporal variation.

In order to check for the applicability of the double wavelet analysis under the conditions of complex nonstationary dynamics with a large number of rhythmic components, let us consider oscillatory processes encountered in the functioning of objects of a living nature. An example is offered by the dynamics of nephrons (structural elements of renal tissue). The results of experimental investigations performed in recent years on rats showed that nephron dynamics contains at least three independent rhythms, which can interact with each other: (i) relatively fast dynamics (5–10 s), (ii) a slower rhythm (30–40 s), and (iii) a very slow rhythmic process (100–200 s). The corresponding oscillatory processes are regular at a normal arterial pressure and become irregular under hypertensive conditions (Figs. 2a and 2b). Experimental dynamics in the latter case is extremely complicated, since the process

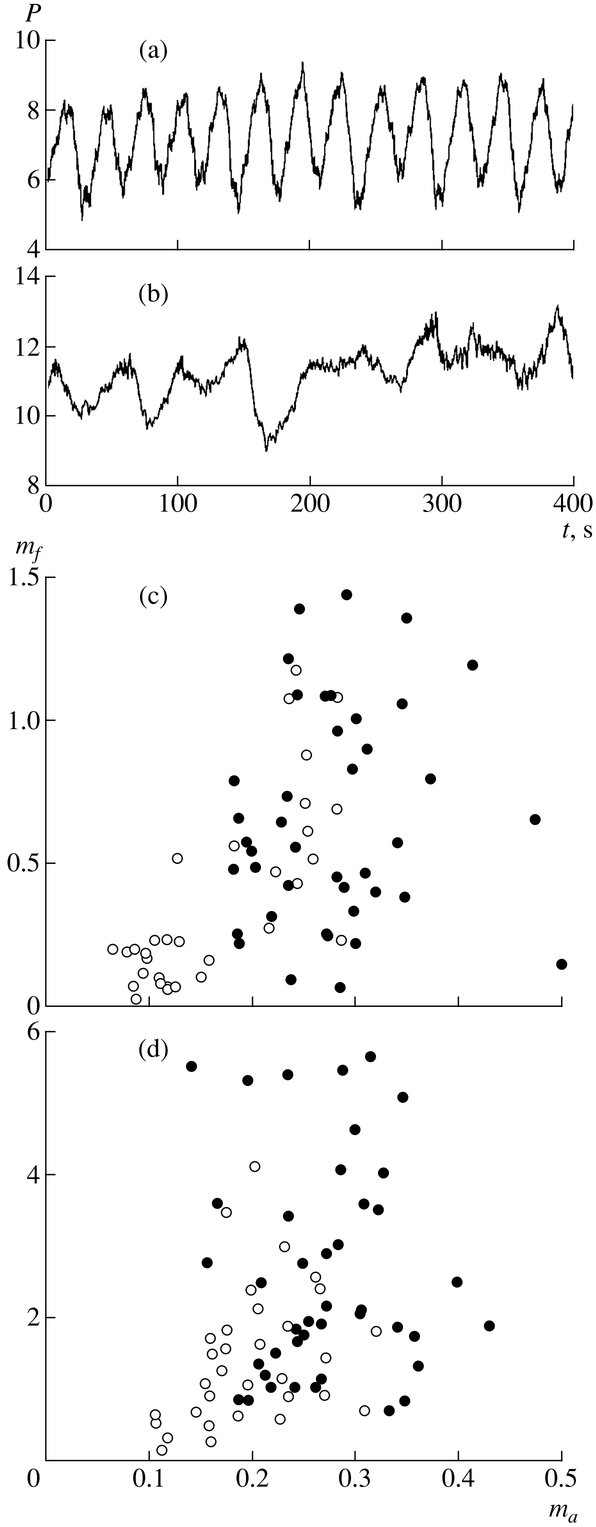

Fig. 2. Typical nephron dynamics: (a, b) time series of signals corresponding to the normal and increase arterial pressure, respectively; (c) the values of amplitude (m_a) and frequency (m_f) modulation of a slow (30–40 s) mode by a very slow (100–200 s) process at normal (open circles) and increased (black circles) arterial pressure; (d) same for the modulation of a fast (5–10 s) dynamics by the slow (30–40 s) mode. Note that the modulation depth is significantly increased under hypertensive conditions.

characteristics exhibit strong variation with time. This example is a good test for the efficiency of the proposed approach based on the double wavelet transform technique. The results of analysis of a rather large experimental data array (about 80 experiments) showed that interactions between the three rhythms in the cases of normal and increased arterial pressure are substantially different. The latter situation is characterized by a stronger interaction of modes, which results in deeper amplitude and frequency modulation. Thus, changes in the regimes of functioning of the living system can be described using the parameters of modulation of oscillatory processes, that is, in well-established terms of radio physics.

The latter system illustrating the functioning of living objects was selected in order to give an example of highly complicated dynamic processes. We have studied possibilities of the double wavelet analysis in a number of other model systems, including the classical cases of amplitude and frequency modulation used in radio engineering (which can be modeled using two harmonic functions), generators of periodic oscillations under external action, etc., (including system with artificially introduced nonstationary elements). All these tests showed coincidence of the data of double wavelet analysis and the anticipated results. In particular, for the above example of a chaotic oscillator (3), a comparison of the modulation depth calculated using the time series of $z(t)$ to the characteristics of $x(t)$ (considered unknown) determined by double wavelet analysis showed good quantitative coincidence.

We believe that the proposed method offers a new powerful tool for investigations into the phenomenon of nonlinear interaction of modes in nonstationary dynamics of various processes irrespective of their nature. This approach can be used for the analysis of interactions involving three (and, probably, more) rhythmic components in radio engineering systems and in various applications of radiophysical techniques.

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