ISSN 1063-7850, Technical Physics Letters, 2008, Vol. 34, No. 4, pp. 306–308. © Pleiades Publishing, Ltd., 2008.
Original Russian Text © A.N. Pavlov, O.N. Pavlova, 2008, published in Pis'ma v Zhurnal Tekhnicheskoĭ Fiziki, 2008, Vol. 34, No. 7, pp. 71–78.

Analysis of Correlation Properties of Random Processes Using Short Signals

A. N. Pavlov* and O. N. Pavlova
Saratov State University, Saratov, 419026 Russia
**e-mail: pavlov@chaos.ssu.runnet.ru*
Received June 26, 2006

Abstract—The possibility of studying the correlation properties of random processes within the framework of a multifractal formalism is considered. It is shown that the calculation of Hölder exponents allows one to judge on the correspondence of an analyzed signal to a process with known statistical properties using a significantly smaller amount of experimental data as compared to that necessary for evaluating the law of correlation decay by means of calculations of the autocorrelation function.

PACS numbers: 05.10.-a, 05.45.Tp

DOI: 10.1134/S1063785008040111

Correlation analysis belongs to the standard methods of investigation of the structure of random processes, which are widely used in solving various applied problems. In addition to the power spectrum and the probability distribution function, the autocorrelation function (ACF) $\Psi(\tau)$ is among the basic characteristics, which are usually calculated in order to judge on the properties of analyzed signals [1–3]. In many applications, it is necessary to know the approximate law of correlation decay—especially for large values of the temporal variable τ—which makes possible the analysis of "long-term memory" in the dynamics of a system generating the signal under consideration. However, the practical implementation of this approach encounters several difficulties. On one hand, the correlation function of a random process that is not periodic (or quasi-periodic) decays to zero and, hence, the ACF calculations for large τ in the case of noisy experimental data can lead to significant errors. Beginning with certain τ, the values of $\Psi(\tau)$ will be comparable with the computational errors and, hence, further calculations become inexpedient. On the other hand, during the processing of signals measured in experiments, it is frequently necessary to obtain estimates based on relatively short time series, which also leads to difficulties related to the possible discrepancy between the ACF calculation using a selected short time series and the theoretically expected result for the infinitely long time series.

In recent years, the problems of correlation analysis have received much attention [4–7]. A special approach to the investigation of long-term correlations has been developed, which is based on the concept of one-dimensional random walk [4, 5]. According to this approach, the decaying ACF is transformed into a certain increasing function, the slope of which characterizes the correlation properties of random processes, including nonstationary ones. The method developed by Peng et al. [4, 5] offers an effective tool for the analysis of long-term correlations; however, it requires a considerable amount of experimental data.

One possible approach to investigations of the correlation properties of random processes using relatively short fragments of their time series is based on the multifractal formalism known as the method of wavelet-transform modulus maxima (WTMM) [8–10]. A specific feature of this approach is that it offers a tool for the "local" investigation of time-dependent functions, while the use of a wavelet transform allows the slow nonstationary variation to be ignored. The efficiency of using the multifractal formalism for a correlation analysis of relatively short time series was briefly considered in [11].

The present Letter is devoted to a more detailed comparison of the WTMM method and the ACF calculations. A detailed description of the multifractal formalism based on the wavelet transform can be found in [9, 11, 12]. This method stipulates description of the statistical properties of a process in terms of the spectrum of singularities $D(h)$, where h is the Hölder's exponent and D is the fractal dimension of a subset of data, which can be characterized by the exponent h. Determination of the singularity spectrum within the framework of the WTMM method is based on the calculation of wavelet transform coefficients (stage 1 of the given algorithm). Then, a partial function $Z(q, a)$ is constructed that represents the sum of local maxima of the moduli of wavelet coefficients with power q on scale a. It is usually assumed [9] that the dependence of $Z(q, a)$ on a has a power character, $Z(q, a) \sim a^{\tau(q)}$, and is described by the quantity $\tau(q)$ called the scaling exponent. By selecting various powers q, it is possible to

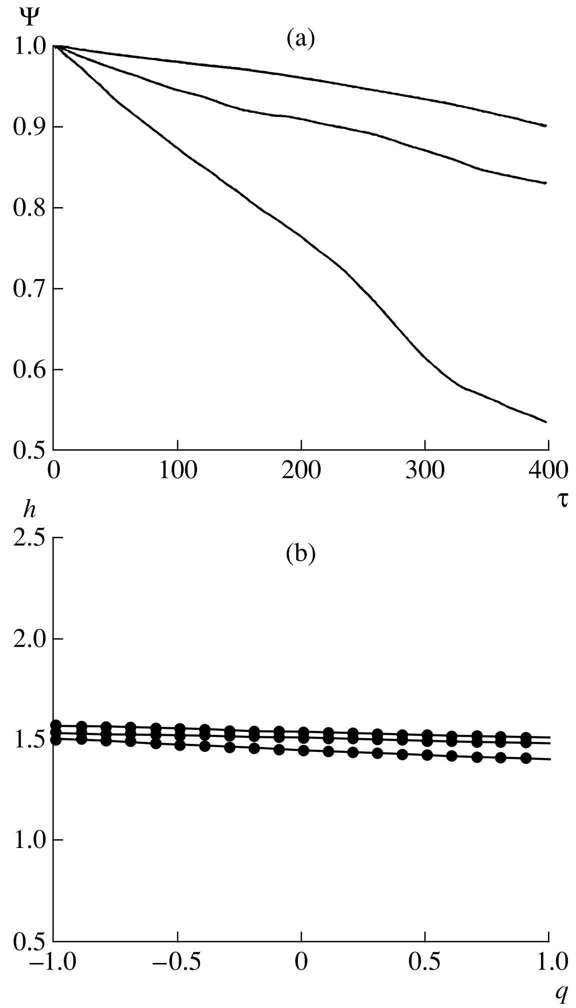

Fig. 1. Comparison of (a) ACFs calculated for three arbitrarily selected time series of the Wiener process with a length 3000 points (τ corresponds to the number of points) and (b) Hölder's exponents calculated for the same time series.

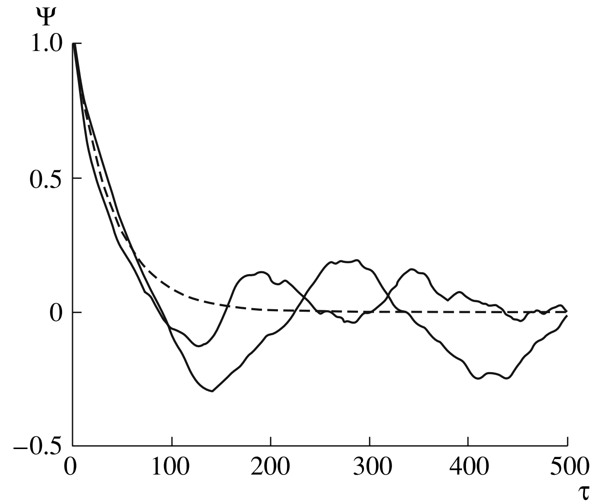

Fig. 2. Comparison of an ACF calculated for two arbitrarily selected time series of the Ornstein–Uhlenbeck process ($\gamma = 1$) with a length 3000 points (solid curve) and a theoretically expected dependence (dashed curve). Note significant deviations of the ACF from the theoretical curve in the region of $\tau > 100$.

obtain a spectrum of Hölder's exponents $h = d\tau(q)/dq$ and calculate the fractal dimensions using the Legendre transform as $D(h) = dh(q) - \tau(q)$. The quantities $h(q)$ describe the scaling of wavelet coefficients along the lines of local maxima and characterize various types of correlated dynamic ($h \neq 0.5$) or the absence of correlations ($h = 0.5$). In particular, $h = 1$ corresponds to the $1/f$ noise, $h = 1.5$ corresponds to a Wiener process, etc. Thus, knowledge of Hölder's exponents allows one to judge on the correspondence of an analyzed signal to a process with known statistical properties. Moreover, estimates based on short signals can be more reliable than the ACF calculations. Indeed, let us consider a Wiener process. ACF calculations (Fig. 1a) show that, in the analysis of relatively short time series, estimates of the law of correlation decay can differ by a factor of 2 for selected time series with a length of 3000 points. The WTMM method predicts the presence of a Wiener process using the same data with a smaller scatter of calculated characteristics. The deviation of the Hölder's exponents from a theoretically predicted value of $h(q) = 1.5$ does not exceed 3–5%.

In order to compare possibilities of the WTMM method and the classical correlation analysis in more detail, let us consider the Ornstein–Uhlenbeck process described by the simple stochastic differential equation

$$\frac{dx(t)}{dt} = \gamma x(t) + \xi(t)$$

which describes the motion of a Brownian particle in a viscous medium under the action of as large number of random impacts [$x(t)$ is the coordinate of the Brownian particle and $\xi(t)$ is the random force]. When the random force is approximated by normal white noise, this process is characterized by an exponentially decaying correlation function: $\Psi(\tau) \sim \exp(-\gamma|\tau|)$. Figure 2 shows the theoretically expected normalized ACF (dashed curve) and the results of $\Psi(\tau)$ calculations from the selected time series with a length of 3000 points. It should be noted that the presence of a relatively short time series leads to significant errors in the attempt to approximate the law of correlation decay using the calculated characteristics. As can be seen from Fig. 2, the variance of estimates of the ACF decay rate is different for the large and small τ values. In the region of relatively long-term correlations (in this example, $\tau > 100$), the ACF calculation using the selected time series leads to significant deviations of the ACF from the theoretical curve.

Figure 3 shows a comparison of the classical correlation analysis and the WTMM method in terms of the variance of estimates of the calculated characteristics. One plot presents the relative error of calculations of the decay rate of ACF and Hölder's exponents versus

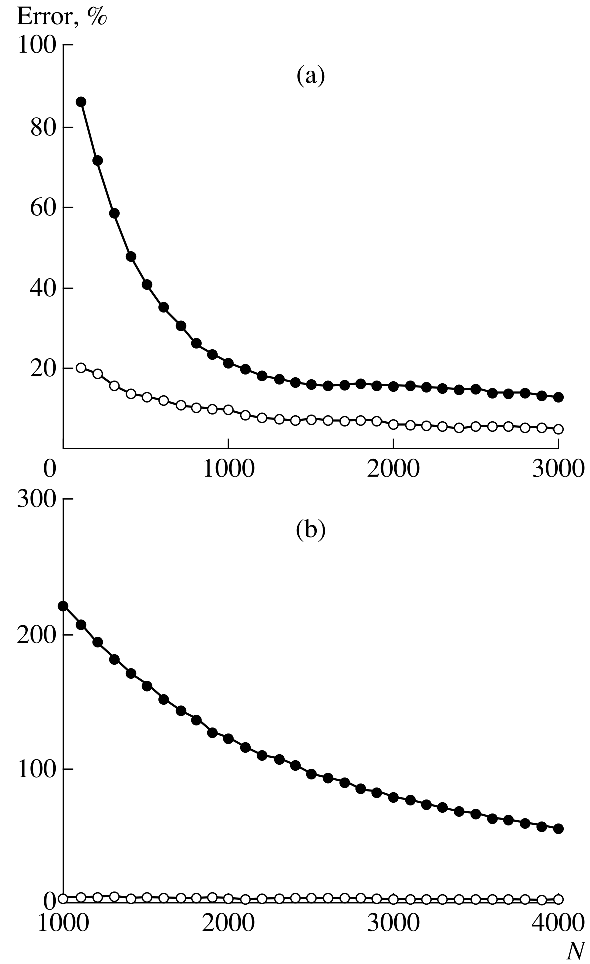

Fig. 3. Plots of (a) the relative error of determining characteristics by the classical correlation analysis (black circles) and WTMM method (open circles) versus the number of points in the analyzed time series for the Ornstein–Uhlenbeck process (averaged over 200 time series) for $\gamma = 1$ in the regions of (a) $\tau < 50$ and (b) $\tau > 100$.

the length of selected time series (the result of averaging over 200 time series of the Ornstein–Uhlenbeck process). As can be seen from Fig. 3a, the WTMM method ensures a smaller variance of the estimates of Hölder's exponents in the case of short-term correlations (from the standpoint of this method, $q < 0$; our calculations were performed for $q \in [-1, -5]$). The advantage of the WTMM method is even more pronounced for the analysis of long-term correlations (Fig. 3b), which corresponds to positive $q \in [1, 5]$. The relative error of calculations of the estimates characteristics in the case of a multifractal analysis is smaller by at least one order of magnitude than that in the case of the classical AFC. Thus, the method of multifractal analysis based on the wavelet transform can be considered as a useful tool for the investigation of correlation properties in cases where a short duration of experimental time series imposes limitations on the reliability of estimates obtained using the standard correlation analysis.

The WTMM method is effective in the analysis of correlations of various lengths, making possible the investigation of the structure of complex signals, for which the law of correlation decay can not be described by a single parameter (exponent). It should be noted, however, that a disadvantage of this approach is the complexity of comparison of the estimated Hölder's exponents and the characteristics commonly accepted in statistical radio physics (correlation time, etc.). Nevertheless, the possibility of establishing a correspondence between the analyzed signal and a process with known statistical properties based on relatively small time series (even in the case of nonstationary data) is of great value in practical research, where the experimental conditions impose limitations on the duration of measured signals.

Acknowledgments. This study was supported in part by the Ministry of Education and Science of the Russian Federation (within the framework of the program "Development of Scientific Potential of Higher School 2006-2008") and the noncommercial Dynasty Foundation.

REFERENCES

1. V. I. Tikhonov, *Statistical Radio Engineering* (Radio i Svyaz', Moscow, 1982) [in Russian].
2. J. Bendat and A. Piersol, *Random Data: Analysis and Measurement Procedures* (John Wiley, New York, 1986).
3. C. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry, and Natural Sciences* (Springer, Heidelberg, 1983).
4. S.-K. Peng, S. Havlin, H. E. Stanley, and A. L. Goldberger, Chaos **5**, 82 (1995).
5. C-K. Peng, S. V. Buldyrev, S. Havlin, et al., Phys. Rev. E. **49**, 1685 (1994).
6. V. S. Anishchenko, T. E. Vadivasova, G. A. Okrokvertskhov, and G. I. Strelkova, Usp. Fiz. Nauk **175**, 163 (2005) [Phys. Usp. **48**, 151 (2005)].
7. V. S. Anishchenko, V. V. Astakhov, A. B. Neiman, et al., in *Nonlinear Dynamics of Chaotic and Stochastic Systems. Tutorial and Modern Development* (Springer, Berlin, 2007).
8. J. F. Muzy, E. Bacry, and A. Arneodo A., Phys. Rev. Lett. **67**, 3515 (1991).
9. J. F. Muzy, E. Bacry, and A. Arneodo, Int. J. Bifurc. Chaos **4**, 245 (1994).
10. P. Ch. Ivanov, L. A. Nunes Amara, A. L. Goldberger, et al., Nature **399**, 461 (1999).
11. A. N. Pavlov and V. S. Anishchenko, Usp. Fiz. Nauk **177**, 859 (2007) [Phys. Usp. **50**, 819 (2007)].
12. A. N. Pavlov, A. R. Ziganshin, and V. S. Anishchenko, Izv. Vyssh. Uchebn. Zaved., Priklad. Nelin. Dinam. **9** (3), 39 (2001).

Translated by P. Pozdeev