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Dynamics of Chaotic and Stochastic Systems

Tutorial and Modern Development

Second edition: improved and enlarged

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To our teachers:

Werner Ebeling, Yuri L. Klimontovich and Frank Moss

Preface to the Second Edition

We present an improved and enlarged version of our book V.S. Anishchenko et al. *Nonlinear Dynamics of Chaotic and Stochastic Systems* published by Springer in 2002. In its basis the new edition of the book corresponds to its first version. When preparing the present edition we made some clarifications in several sections as well as corrected the noticed misprints in formulas. Besides, three new sections were added to Chapter 2. They are "*Statistical Properties of Dynamical Chaos*", "*Effects of Synchronization in Extended Self-Sustained Oscillatory Systems*", and "*Synchronization in Living Systems*". The indicated sections naturally enroll in the material of the second chapter and reflect the most interesting results obtained by the authors after publication of the first edition.

We hope that the new edition of the book will be of great interest for a wide auditorium of readers who are already specialists or only beginning researchers in the fields of nonlinear oscillation and wave theory, dynamical chaos, synchronization and stochastic process theory.

Saratov, Berlin, and St. Louis
April 2006

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Preface to the First Edition

This book is devoted to the classical background and to contemporary results on nonlinear dynamics of deterministic and stochastic systems. Considerable attention is given to the effects of noise on various regimes of dynamical systems with noise-induced order.

On the one hand, there exists a rich literature of excellent books on nonlinear dynamics and chaos; on the other hand, there are many marvelous monographs and textbooks on the statistical physics of far-from-equilibrium and stochastic processes. This book is an attempt to combine the approach of nonlinear dynamics based on the deterministic evolution equations with the approach of statistical physics based on stochastic or kinetic equations. One of our main aims is to show the important role of noise in the organization and properties of dynamic regimes of nonlinear dissipative systems.

We cover a limited region in the interesting and still expanding field of nonlinear dynamics. Nowadays the variety of topics with regard to deterministic and stochastic dynamic systems is extremely large. Two main criteria were followed in writing the book and to give a reasonable and closed presentation: (i) the dynamic model should be minimal, that is, most transparent in the physical and mathematical sense, and (ii) the model should be the simplest which nevertheless clearly demonstrates most important features of the phenomenon under consideration.

The book consists of three chapters. The first chapter serves as a brief introduction, giving the fundamental background of the theory of nonlinear deterministic and stochastic systems and a classical theory of the synchronization of periodic oscillations. All basic definitions and notions necessary for studying the subsequent chapters without referring to special literature are presented.

The second chapter is devoted to deterministic chaos. We discuss various scenarios of chaos onset including the problem of the destruction of two- and three-frequency quasiperiodic motion. Different aspects of synchronization and chaos control as well as the methods of reconstruction of attractors and dynamic systems from experimental time series are also discussed.

The third chapter is concerned with stochastic systems whose dynamics essentially depend on the influence of noise. Several nonlinear phenomena are discussed: stochastic resonance in dynamic systems subjected to harmonic and complex signals and noise, stochastic synchronization and stochastic ratchets, which are the noise-induced ordered and directed transport of Brownian particles moving in bistable and periodic potentials. Special attention is given to the role of noise in excitable dynamics.

The book is directed to a large circle of possible readers in the natural sciences. The first chapter will be helpful for undergraduate and graduate students in physics, chemistry, biology and economics as well as for lecturers of these fields interested in modern problems of nonlinear dynamics. Specialist of nonlinear dynamics may use this part as an extended dictionary. The second and the third chapters of the book addresses to specialists in the field of mathematical modelling of the complex dynamics of nonlinear systems in the presence of noise.

We tried to write this book in such a manner that each of the three chapters can be understood in most parts independently of the others. Particularly, each chapter has its own list of references. This choice is based on the desire to be helpful to the reader. Undoubtedly, the lists of references are incomplete. We indicate the most important contributions on each particular subject studied and give references to papers whose results were directly used in writing the book.

This book is a result of the long-term collaboration of the Nonlinear Dynamics Laboratory at Saratov State University, the group of Applied Stochastic Processes of Humboldt University at Berlin, and the Center for Neurodynamics at the University of Missouri at St. Louis. We want to express our deep gratitude to W. Ebeling, Yu.L. Klimontovich and F. Moss for their support, scientific exchange and constant interest. We acknowledge fruitful discussions with C. van den Broeck, P. Hänggi, J. Kurths, A. Longtin, A. Pikovski, and Yu.M. Romanovski. The book has benefited a lot from our coauthors of the original literature. We wish to thank A. Balanov, R. Bartussek, V. Bucholtz, I. Diksthein, J.A. Freund, J. Gárcia-Ojalvo, M. Hasler, N. Janson, T. Kapitaniak, I. Khovanov, M. Kostur, P. S. Landa, B. Lindner, P. McClintock, E. Mosekilde, A. Pavlov, T. Pöschel, D. Postnov, P. Reimann, R. Rozenfeld, P. Ruszczynsky, A. Shabunin, B. Shulgin, U. Siewert, A. Silchenko, O. Sosnovtseva, A. Zaikin and C. Zülicke for regular and fruitful discussions, criticism and valuable remarks which give us deeper insight into the problems we study.

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